UiO : Fysisk institutt

Det matematisk-naturvitenskapelige fakultet

Lecture 11

This week

- **Wednesday:** Relativistic four-vectors, Lorentz transformations and Minkowski diagrams. (Sections 4.3, 4.4, and 4.5)
- **Thursday:** Problem set 5 with last of Lagrangian and Hamiltonian for now. Includes mid-term exam question from 2008 (famous physics problem).
- **Friday:** Length contraction, time dilatation and proper time. (Sections 5.1, 5.2, and 5.3)

  Also twin paradox (Section 5.4) if time.

Recap

- A **reference frame** or **rest frame** (RF) is a coordinate system with origin and orientation.
- An **inertial RF** is a non-accelerated RF.
- Relativity (Galilean) says that a particle with velocity u and u' in two RFs S and S' with relative velocity v has u' = u-v under
$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t = t$$
- To have the same velocity of light in all frames we must introduce **Lorentz transformations**
$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Recap

- We define an **invariant distance (metric)**

$$\Delta s^2 \equiv c^2 \Delta t^2 - \Delta x^2$$

which is the same in all RFs.

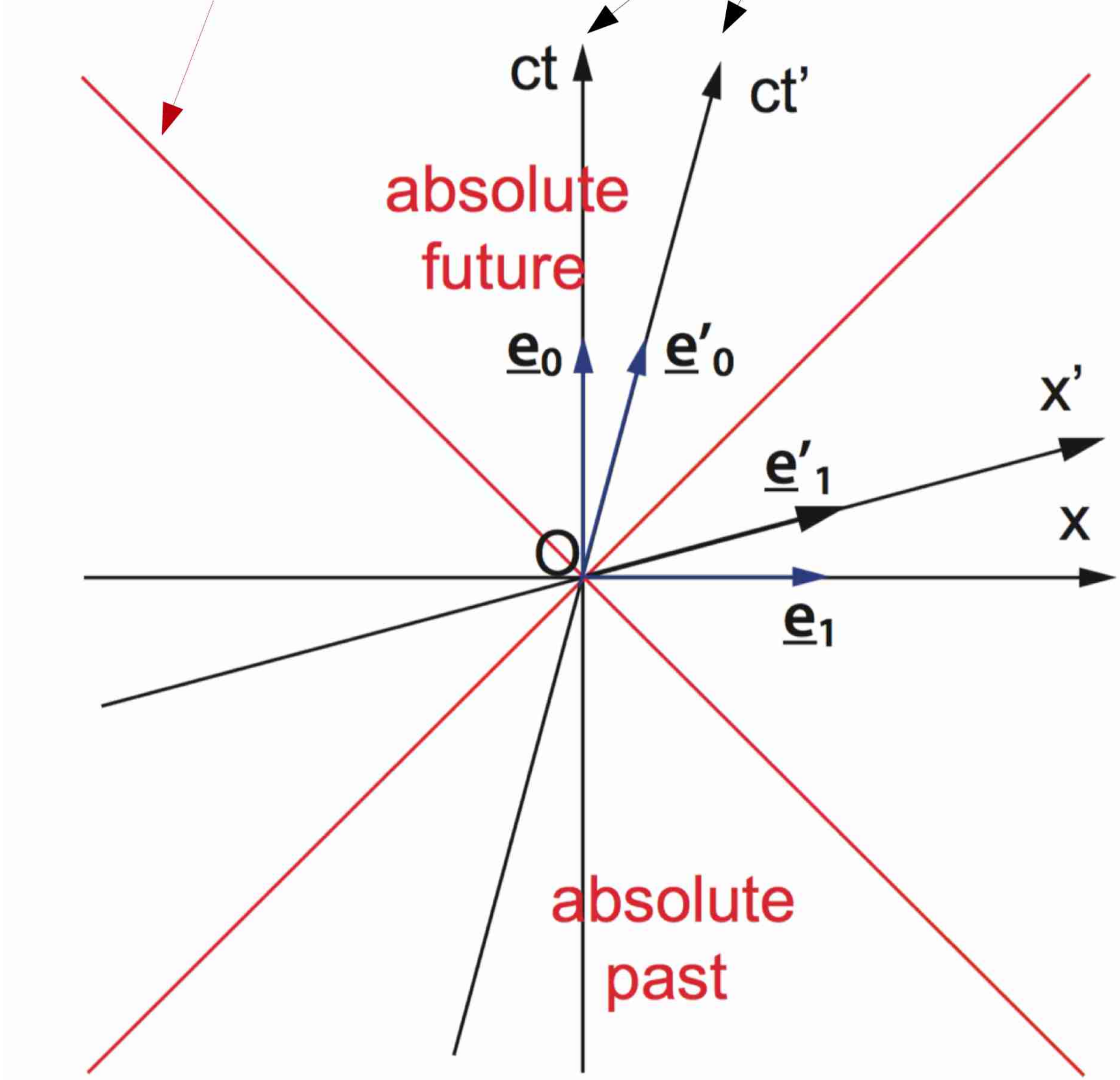
- A crucial point of notation is **four-vectors**

$$x = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}, \text{ in components } x^\mu \text{ where } \mu = 0,1,2,3$$

Today

- Four-vectors
  - Essential tool for all relativistic physics.
- More general (Lorentz) transformations
  - All possible symmetries of special relativity.
- Minkowski diagrams
  - Keeping track of causal relations between space-time points.

Minkowski diagram

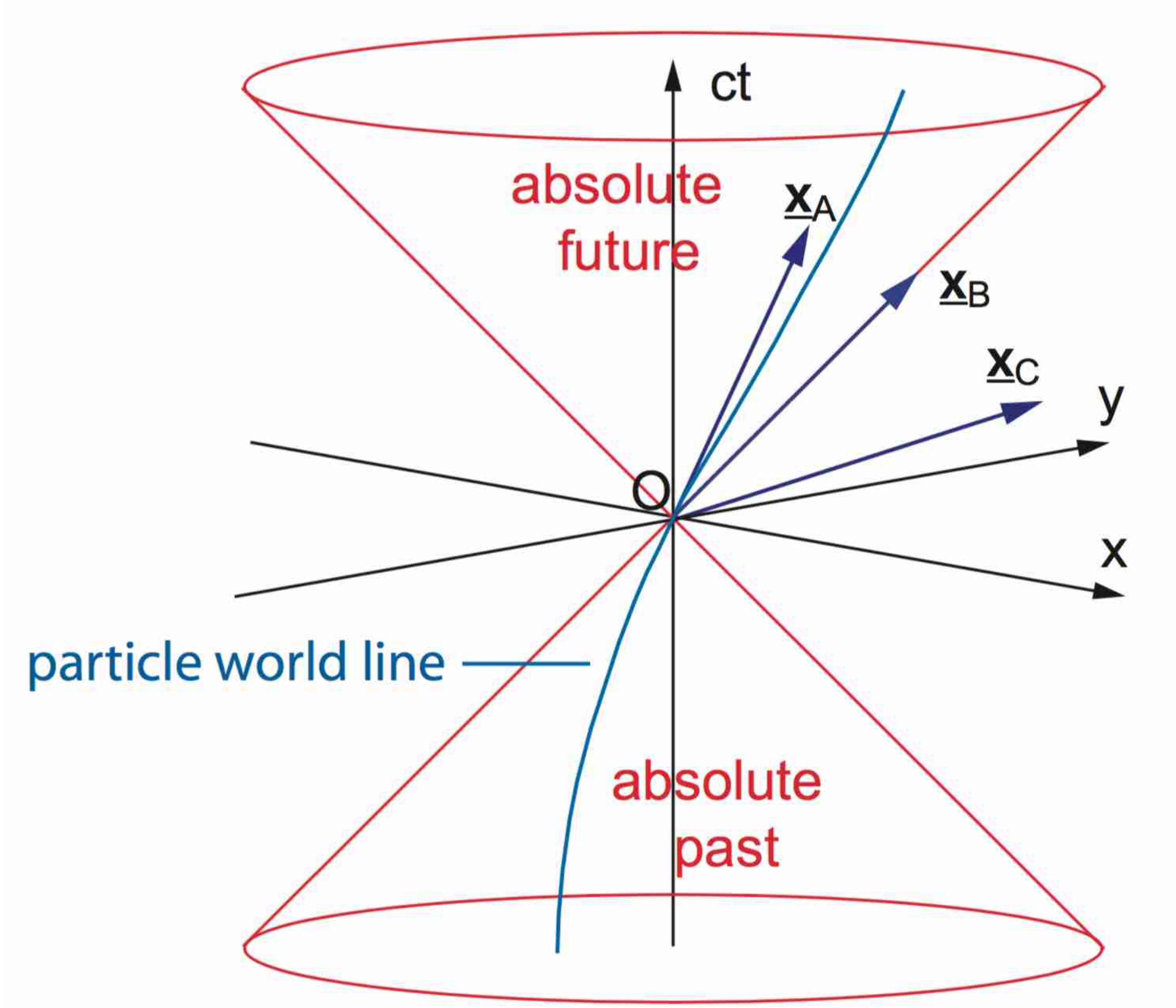


Two RFs S and S' with relative velocity β = v/c

$$x' = \gamma\left(x - \beta ct\right)$$

$$ct' = \gamma\left(ct - \beta x\right)$$

Minkowski diagram – light cone



Two RFs S and S' with relative velocity β = v/c

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Summary

- We can write **Lorentz transformations** as the matrix multiplication

$$x'^{\mu} = L^{\mu}{}_{\nu} x^{\nu}$$

or x' = Lx, where, for a boost in the x-direction,

$$L = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Adding translations we have the **Poincaré transformation** x' = Lx+a.