UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 16

This week

- **Wednesday:** Hyperbolic motion (Spaceship Navigation 101). Relativistic energy and momentum, the energy-momentum relation. (Sections 7.1.1, 7.2 and 7.3)
- **Thursday:** Problem set 8, relativistic mechanics and space travel.
- **Friday:** Doppler effect with photons, relativistic scattering. (Sections 7.4-7.7)

Recap

- We define four-velocity and four-acceleration in terms of the proper time τ

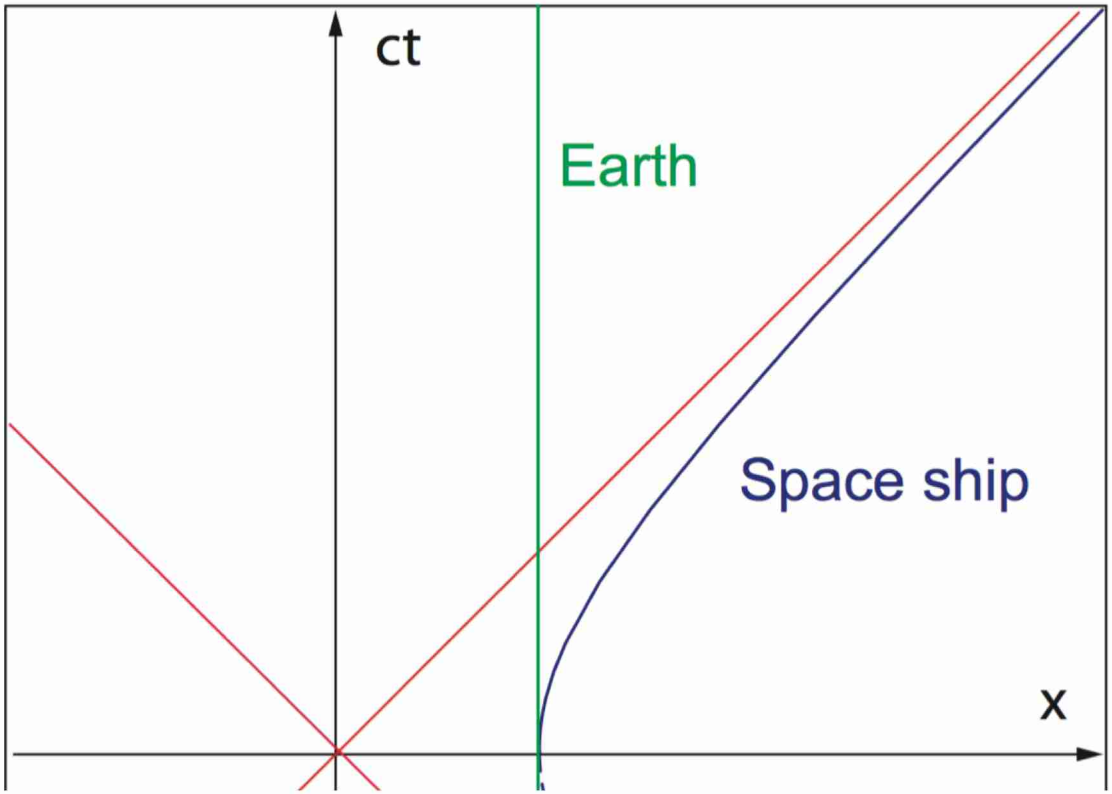
$$U^\mu \equiv \frac{dx^\mu}{d\tau}; \qquad A^\mu \equiv \frac{d^2x^\mu}{d\tau^2}$$

- The **proper acceleration** is defined as the acceleration in the instantaneous inertial RF (the changing RF where v = 0).
  - This is the acceleration measurable with an accelerometer (so not free fall).

Today

- Spaceship Navigation SF101
  - Velocity, position and time for a spaceship.
  - Hyperbolic motion.
  - A very very strange constant.
- Relativistic energy and momentum
  - The energy-momentum relation.
- Spaceship economics SF102
  - The practical issues in theoretical spaceship travel.

Spaceship Minkowski diagram



τ	1 y	2 y	3 y	5 y	7 y	11 y	15 y
t	1.2 y	3.6 y	10.0 y	74 y	548 y	30 000 y	$1.6 \cdot 10^6$ y
$x - x_0$	0.5 ly	2.8 ly	9.1 ly	73 ly	547 ly	30 000 ly	$1.6 \cdot 10^6$ly

Summary

- Relativistic four-momentum p^μ is defined as

  $$p^\mu = mU^\mu = (\gamma mc, \gamma m\vec{v}) = (E/c, \vec{p})$$

  where E and p is the relativistic energy and momentum. These reduce to ordinary kinetic energy plus rest energy, and to ordinary momentum in the non-relativistic limit.

- These lead to the energy-momentum relation

  $$E^2 = p^2c^2 + m^2c^4$$

  which allows massless particles with E = pc.