UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 19

Recap

- Relativistic four-force K^μ is defined as

$$K^\mu = \frac{dp^\mu}{d\tau} = \gamma\left(\frac{1}{c}\vec{c}\cdot\vec{F},\, \vec{F}\right)$$

where the relativistic force F is

$$\vec{F} = \frac{d\vec{p}}{dt}, \qquad \vec{p} = \gamma m \vec{v}$$

- We can write the Lorentz force (force from electromagnetic field) on covariant form as

$$K^\mu = eF^{\mu\nu}U_\nu \qquad \left(F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu\right)$$

where $F^{\mu\nu}$ is the electromagnetic field strength.

Today

- A covariant Lagrangian
  - The Lagrangian in terms of Lorentz vectors only.
  - Lagrange's equation on covariant form.
  - Example with free particle.
  - Example with free particle.
  - Example with free particle.
  - Example with particle in e.m. field.

Summary

- We can write a relativistic action as

$$S = \int_{\tau_1}^{\tau_2} L\left(x^\mu(\tau), U^\mu(\tau), \tau\right) d\tau$$

which is a Lorentz (transformation) invariant scalar using a Lorentz invariant Lagrangian L.

- Lagrange's equations are then

$$\frac{d}{d\tau}\frac{\partial L}{\partial U^\mu} - \frac{\partial L}{\partial x^\mu} = 0$$

where τ is the proper time and U is the four-velocity.