UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 22

This week

- **Wednesday:** Lorentz transformations of electric and magnetic fields. (Sections 9.8-9.9)
- **Thursday:** Problem set 10. (Questions on Maxwell's equations & exam question from 2013 on relativity.)
- **Friday:** Electromagnetic waves, polarization. (Sections 10.1 and 10.2)

Recap

- We can write Maxwell's equations on covariant form as

$$\partial_\mu F^{\mu\nu} = \mu_0 j^\nu$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

is the **electromagnetic field strength tensor**,

$$j^\nu = (c\rho, \vec{j})$$

is the **four-vector current density**, and

$$A^\mu = (\frac{1}{c}\phi, \vec{A})$$

is the **four-potential**.

Recap

- The explicit form of $F^{\mu\nu}$ is

$$F^{\mu\nu} = \begin{bmatrix} 0 & -\frac{1}{c}E_1 & -\frac{1}{c}E_2 & -\frac{1}{c}E_3 \\ \frac{1}{c}E_1 & 0 & -B_3 & B_2 \\ \frac{1}{c}E_2 & B_3 & 0 & -B_1 \\ \frac{1}{c}E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

and we also define the **dual tensor**

$$\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

Today

- Lorentz transformations of e.m. fields
  - Transformation of the electromagnetic field strength tensor
  - What happens to the electric and magnetic fields?
  - An example
- Lorentz invariants for the electromagnetic fields
- Long example: current in a wire in different reference frames.

Mid-term evaluation

- Problematic difference in notation between lectures and lecture notes.
- Difficult to see connections between the use of tensors in different courses.
- More problems requested (non-mandatory).
- Virtual work WTF.
- Mid-term – more of a test of how many friends you have than how much you have learnt.

Summary

- Under Lorentz transformations, E- & B-field components parallel and perpendicular to boost transform as

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel}, \quad \vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \vec{v}\times\vec{B})$$

$$\vec{B}'_{\parallel} = \vec{B}_{\parallel}, \quad \vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \frac{1}{c^2}\vec{v}\times\vec{B})$$

- We can form the following Lorentz invariants from the electromagnetic field strength tensor

$$\frac{1}{2}F^{\mu\nu}F_{\mu\nu} = \vec{B}^2 - \frac{1}{c^2}\vec{E}^2, \quad \frac{1}{4}F^{\mu\nu}\widetilde{F}_{\mu\nu} = -\frac{1}{c}\vec{E}\cdot\vec{B}$$