UiO : Fysisk institutt

Det matematisk-naturvitenskapelige fakultet

Lecture 23

Recap

- Under Lorentz transformations, E- & B-field components parallel and perpendicular to boost transform as

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel}, \quad \vec{E}'_{\perp} = \gamma\left(\vec{E}_{\perp} + \vec{v}\times\vec{B}\right)$$

$$\vec{B}'_{\parallel} = \vec{B}_{\parallel}, \quad \vec{B}'_{\perp} = \gamma\left(\vec{B}_{\perp} - \frac{1}{c^2}\vec{v}\times\vec{B}\right)$$

- We can form the following Lorentz invariants from the electromagnetic field strength tensor

$$\frac{1}{2}F^{\mu\nu}F_{\mu\nu} = \vec{B}^2 - \frac{1}{c^2}\vec{E}^2, \quad \frac{1}{4}F^{\mu\nu}\tilde{F}_{\mu\nu} = \frac{1}{c}\vec{E}\cdot\vec{B}$$

Recap

- We can write Maxwell's equations on covariant form as

$$\partial_\mu F^{\mu\nu} = \mu_0 j^\nu$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

- In the Lorentz gauge

$$\partial_\mu A^\mu = 0$$

this reduces to

$$\partial^\mu \partial_\mu A^\nu = j^\nu$$

Today

- Electromagnetic waves
  - Focus on plane waves (monochromatic waves)
  - Description by e.m. potential
  - Description in terms of E- and B-fields.
- Polarization of electromagnetic waves
  - Linear
  - Elliptic (general)

Mid-term evaluation

- Problematic difference in notation between lectures and lecture notes.
- Difficult to see connections between the use of tensors in different courses.
- More problems requested (non-mandatory).
- Virtual work WTF.
- Mid-term – more of a test of how many friends you have than how much you have learnt.

Summary

- In terms of e.m. potential the free (no source) plane wave solution of Maxwell's equations is

$$\vec{A}(\vec{r},t) = \vec{A}_0 e^{i(\vec{k}\vec{r}-\omega t)}$$

where the wave (number) vector k fulfils

$$\vec{k}\cdot\vec{A}_0 = 0$$

- The most general mode of polarization is **elliptic polarization** where the E-field rotates in an ellipse in the plane transverse to the wave vector. The E- and B-fields are perpendicular.