UiO : Fysisk institutt

Det matematisk-naturvitenskapelige fakultet

# Lecture 24

# This week

- **Wednesday:** Electromagnetic energy and momentum density. (Section 10.3)
- **Thursday:** Problem set 11. (More electromagnetism. Please check that you have enough problems sets to take the exam!)
- **Friday:** Electrostatic equation and multipole expansion. (Section 11.1)

# Recap

- In terms of e.m. potential the free (no source) plane wave solution of Maxwell's equations is
$$\vec{A}(\vec{r},t) = \vec{A}_0 e^{i(\vec{k}\vec{r}-\omega t)}$$
where the wave (number) vector k fulfils
$$\vec{k}\cdot\vec{A}_0 = 0$$
- The electric and magnetic fields are (in Coulomb gauge) given as
$$\vec{E} = i\omega\vec{A}, \quad \vec{B} = i\vec{k}\times\vec{A}$$
and related through the unit vector in k-direction
$$\vec{E} = -c\vec{n}\times\vec{B}, \quad \vec{B} = \frac{1}{c}\vec{n}\times\vec{E}$$

# Today

- The energy and momentum in an electromagnetic field
  - We derive the energy density and the momentum density.
  - Along the way Poynting's vector (the energy current density) appears.
  - The results allow us to define the energy-momentum tensor for a field.
  - Two examples:
    - Energy & momentum of a plane wave
    - Energy of a point particle (HELP!)

# Summary

- The energy current density S (**Poynting's vector**) and the energy density u is

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}, \quad u = \frac{1}{2}\left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right)$$

- The **energy-momentum tensor** for electromagnetic fields is defined as

$$T^{\mu\nu} = \frac{1}{\mu_0}\left(-F^{\mu\rho}F^{\nu}{}_{\rho} + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}\right)$$

this contains $T^{00} = u$ and $T^{0i} = S_i/c$.