UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 25

Recap

- In terms of e.m. potential the free (no source) plane wave solution of Maxwell's equations is
  $$\vec{A}(\vec{r},t) = \vec{A}_0 e^{i(\vec{k}\vec{r}-\omega t)}$$
  where the wave (number) vector k fulfils
  $$\vec{k}\cdot\vec{A}_0 = 0$$
- The electric and magnetic fields are (in Coulomb gauge) given as
  $$\vec{E} = i\omega\vec{A}, \quad \vec{B} = i\vec{k}\times\vec{A}$$
  and related through the unit vector in k-direction
  $$\vec{E} = -c\vec{n}\times\vec{B}, \quad \vec{B} = \frac{1}{c}\vec{n}\times\vec{E}$$

Recap

- The energy current density S (**Poynting's vector**) and the energy density u is

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}, \quad u = \frac{1}{2}\left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right)$$

  The momentum g density is proportional to S

$$\vec{g} = \vec{S}/c^2$$

- The **energy-momentum tensor** for electromagnetic fields is defined as

$$T^{\mu\nu} = \frac{1}{\mu_0}\left(-F^{\mu\rho}F^{\nu}{}_{\rho} + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}\right)$$

  This contains $T^{00} = u$ and $T^{0i} = S_i/c$.

Today

- Potential and fields from static sources.
  - Electrostatics (static electric charge) [today]
  - Magnetostatics (constant current) [next week]
- General solution for electrostatics
  - Easy to write down, difficult to calculate
- Approximate solution at large distance
  - Multipole expansion

Summary

- The electrostatic solution for the potential is

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r})}{|\vec{r}-\vec{r}\,'|}\, d^3\vec{r}\,'$$

- At large distances from the charges this can be approximated in the multipole expansion

$$\rho(\vec{r}) = \rho_0(\vec{r}) + \rho_1(\vec{r}) + \rho_2(\vec{r}) + \ldots$$

with the monopole and dipole contributions

$$\rho_0(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r}, \quad \rho_1(\vec{r}) = \frac{\vec{r}\cdot\vec{p}}{4\pi\epsilon_0 r^3}$$

where p is the dipole moment $\vec{p} = \int \rho(\vec{r})\vec{r}\, d^3\vec{r}\,'$