UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 27

Recap

- The **magnetostatic solution** for the potential is
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r})}{|\vec{r}-\vec{r}\,'|}\, d^3\vec{r}\,'$$
- At large distances from the current this can be approximated in the **multipole expansion**
$$\vec{A}(\vec{r}) = \vec{A}_0(\vec{r}) + \vec{A}_1(\vec{r}) + \vec{A}_2(\vec{r}) + \dots$$
where the **monopole** contribution is $A_0 = 0$.
- The force and torque from external fields are
$$\vec{F}_e = Q\vec{E} + (\vec{p}\cdot\vec{\nabla})\vec{E} + \dots, \quad \vec{\tau}_e = \vec{p}\times\vec{E} + \dots$$
$$\vec{F}_m = (\vec{m}\cdot\vec{\nabla})\vec{B} + \dots, \quad \vec{\tau}_m = \vec{m}\times\vec{B} + \dots$$

Today

- Solution to time dependent sources
  - Green's functions (light).
  - Fourier transformation to Helmholtz' equation.
  - Solution in terms of retarded sources.

Summary

- The scalar and vector potential for time-dependent sources is

$$\phi(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r},t_-)}{|\vec{r}-\vec{r}\,'|}\, d^3\vec{r}\,'$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r},t_-)}{|\vec{r}-\vec{r}\,'|}\, d^3\vec{r}\,'$$

where the time for the source is the retarded time

$$t_- = t - |\vec{r}-\vec{r}\,'|/c$$