Lecture 29

Recap

- The scalar and vector potential for time-dependent sources is

$$\phi(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r},t_-)}{|\vec{r}-\vec{r}\,'|}\, d^3\vec{r}\,'$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r},t_-)}{|\vec{r}-\vec{r}\,'|}\, d^3\vec{r}\,'$$

where the time for the source is the **retarded time**

$$t_- = t - |\vec{r}-\vec{r}\,'|/c$$

Recap

- Radiation fields far from a source are (usually) dominated by terms that go as 1/r

$$\vec{B}_{\text{rad}}(\vec{r},t) = \frac{\mu_0}{4\pi rc}\left(\ddot{\vec{p}}\times\hat{n} + \frac{1}{c}(\ddot{\vec{m}}\times\hat{n}) + \frac{1}{2}c\,\dddot{\vec{D}}_{\hat{n}}\times\hat{n} + \ldots\right)_{\text{ret}}$$

$$\vec{E}_{\text{rad}}(\vec{r},t) = -c\,\hat{n}\times\vec{B}_{\text{rad}}(\vec{r},t)$$

- Here n is a unit vector in the direction of observation, while the **electric dipole moment** p and **magnetic dipole moment** m are given by

$$\vec{p} = \int \vec{r}\rho(\vec{r},t)d^3\vec{r}, \quad \vec{m} = \frac{1}{2}\int \vec{r}\times\vec{j}(\vec{r},t)d^3\vec{r}$$

Today

- More physical discussion of radiation fields
  - When is the approximation valid?
  - When can it be simplified further?
- Electric dipole radiation (E1)
  - Poynting's vector
  - Power radiated
  - Example with antenna
- Larmor's formula
  - Radiation from non-relativistic point charge

Summary

- The radiation field description applies when r >> a and r >> λ, where a is the size of the source and λ the wavelength of radiation.
- When λ >> a the dipole contributions are dominant.
- Larmor's formula gives the power radiated by a non-relativistic point charge q with acceleration a at a distance r as

$$P(t) = \frac{\mu_0 q^2}{6\pi^2 c} a_{\text{ret}}^2 \qquad t_- = t - |\vec{r} - \vec{r}(t_-)|/c$$