UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

# Lecture 3

# This week

- **Wednesday:** Derivation of Lagrange's equation. (Section 2.1)
- **Thursday:** Problem set 1 (topic: generalized coordinates and virtual displacement)
- **Friday:** Symmetries of the Lagrangian. (Sections 2.2-2.3.4)
- My plan is to skip Section 2.2 for the time being. (Basically harmonic oscillator.)

# Today

- Proof of Lagrange’s equation.
  - Same approach as for static equilibrium condition.
- As many examples of using Lagrange’s equation as we have time for.

# Recap

- We define generalized coordinates $q_j$ from the original coordinates $r_i$ by using the constraints.
- The virtual displacement is the displacement of the original coordinates $r_i$ by a change in the generalized coordinates $q_j$ at fixed time.

$$\delta \vec{r}_i = \sum_{j=1}^{d} \frac{\partial \vec{r}_i}{\partial q_j} \delta q_j$$

- Constraint forces are the forces resulting from applied forces and the enforced constraints.

# Recap

- Static equilibrium can be reformulated through the principle of no virtual work for applied forces

$$\delta W = \sum_i \vec{F}_i^a \cdot \delta \vec{r}_i = 0$$

- Or, as extremal point in the potential energy expressed in generalized coordinates

$$\frac{\partial V}{\partial q_j} = 0$$

- This uses generalized forces

$$F_j = \sum_i \vec{F}_i^a \cdot \frac{\partial \vec{r}_i}{\partial q_j}$$

# Summary

- With the Lagrange function

$$L(q,\dot{q},t) = K(q,\dot{q},t) - V(q,t)$$

we can find the equations of motion for the system from Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$