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# Lecture 6

# Recap

- If the Lagrangian L has no explicit time dependence
$$\frac{\partial L}{\partial t} = 0,$$
then the Hamiltonian
$$H \equiv \sum_i \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L,$$
is a constant of motion/conserved quantity.
- When constraints are time-independent (and holonomic) the Hamiltonian is given by H = K+V.

# Recap

- Total time derivative terms in the Lagrangian do not change the e.o.m.
- For velocity dependent potentials (forces) the generalized force is written as

$$F_j = \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_i}\right) - \frac{\partial U}{\partial q_i}$$

leading to the Lagrange function L = K – U in Lagrange's equation.

# Plan for today

- Charged particle in an electromagnetic field. (Section 2.5)
  - Electromagnetic potentials instead of fields (short preview of Section 9.5).
  - The force on a free charged particle in terms of a velocity dependent “potential” U.
  - The Lagrangian.
  - The conjugate momentum.
  - The Hamiltonian.
  - A symmetry of the Lagrangian – gauge invariance.

# Summary

- The electromagnetic (e.m.) potentials give a potential energy $U = e\phi - e\vec{v}\cdot\vec{A}$
- The conjugate momentum is
$$\vec{p} = m\vec{v} + e\vec{A}$$
and the Hamiltonian
$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 + e\phi$$
- The e.o.m. are invariant under the gauge transformations
$$\phi \rightarrow \phi' = \phi - \frac{\partial\chi}{\partial t}, \qquad \vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla}\chi$$