UiO : Fysisk institutt

Det matematisk-naturvitenskapelige fakultet

Lecture 7

This week

- **Wednesday:** Hamilton's equations, Hamiltonian for charged particles in an electromagnetic field. (Sections 3.1 and 3.2)
- **Thursday:** Problem set 3 (main topic: even more e.o.m. from Lagrange’s equation, cyclic coordinates and how to remove them)
- **Friday:** Application of Hamilton's equations, phase space. (Sections 3.2.1, 3.3, and 3.4)

Today

- Hamilton’s equations
  - Derivation from Lagrange’s equations
  - Simple example with 1D harmonic oscillator
  - Once more (with feeling) the charged particle in magnetic field example
  - Introducing (?) the Levi-Civita symbol (The Horror! The Horror!)

Recap

- The Hamiltonian H is given by

$$H \equiv \sum_i \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L = \sum_i p_i \dot{q}_i - L$$

- Here the generalized/conjugate/canonical momenta are

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

- By solving the velocity in terms of p_i we can write H as a function of q_i and p_i (canonical position and momenta).

Recap

- The e.m. potentials give a potential energy

$$U = e\phi - e\vec{v}\cdot\vec{A}$$

- The conjugate momentum is

$$\vec{p} = m\vec{v} + e\vec{A}$$

and the Hamiltonian

$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 + e\phi$$

- The resulting e.o.m. are invariant under the gauge transformations

$$\phi \rightarrow \phi' = \phi - \frac{\partial \chi}{\partial t}, \qquad \vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla}\chi$$

Summary

- Expressing the Hamiltonian H in terms of the canonical position and momentum q_i and p_i the e.o.m can be written as Hamilton's equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}, \qquad i = 1, \ldots, d$$

- To simplify calculations it is useful to introduce the totally antisymmetric Levi-Civita symbol

$$\epsilon_{ijk} \equiv \begin{cases} 1 & \text{for } i \neq j \neq k \text{ and cyclic} \\ -1 & \text{for } i \neq j \neq k \text{ and not cyclic} \\ 0 & \text{otherwise} \end{cases}$$