

FYS3120 Classical Mechanics and Electrodynamics

Discussion 1

January 26, 2025

Problem 1 Give definitions or explanations of, the following terms:

- a) Constraints.
- b) Degrees of freedom.
- c) Non-holonomic constraints. Give two examples.
- d) Generalised coordinates.
- e) Virtual displacement.

Problem 2 Count the (holonomic) constraints and determine the degree of freedom for:

- a) A football constrained to lie on the football field.
- b) A rigid body in three dimensions.
- c) A rigid body in two dimensions.
- d) A human head.
- e) Human arm.

Problem 3 Generalised forces

In this problem we shall start with single particle with a force acting on it. The position vector of the particle is

$$\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z = \sum_{i=1}^3 x_i\mathbf{e}_i,$$

and the applied force is

$$\mathbf{F}(x, y, z) = F_x\mathbf{e}_x + F_y\mathbf{e}_y + F_z\mathbf{e}_z = \sum_{i=1}^3 F_i\mathbf{e}_i.$$

When each component of the position vector, x_i , is a function of a set of generalised coordinates $q = \{q_1, \dots, q_d\}$, the transformation to the j -th component of the generalised force is given by

$$\mathcal{F}_j = \mathbf{F} \cdot \frac{\partial \mathbf{r}}{\partial q_j} = \sum_{i=1}^3 F_i \frac{\partial x_i}{\partial q_j}.$$

- a) We constrain the particle to move in the xy -plane. Determine the constraint equation.

- b) We introduce generalised coordinates $q = \{q_1, q_2\}$ such that

$$\begin{aligned}x(q) &= q_1 f(q_2), \\y(q) &= q_1 g(q_2),\end{aligned}$$

where f and g are smooth (infinitely differentiable¹) functions of q_2 . Determine the components of the generalised force.

- c) Let $f(q_2) = \cos q_2$ and $g(q_2) = \sin q_2$. Determine $\mathcal{F}_1$ and $\mathcal{F}_2$. What is the physical interpretation of $\mathcal{F}_1$ and $\mathcal{F}_2$?

¹As you may know, this is a quite common assumption in physics, so we often do not bother stating it.