

FYS3120 Classical Mechanics and
Electrodynamics

Discussion 6

March 3, 2025

Problem 1

- a) What is proper time?
- b) How do we distinguish time-like, space-like, and light-like four-vectors?
- c) What are contravariant and covariant tensors?
- d) In a Minkowski diagram, what should the angle between the light-cone and the ct -axis be?

Problem 2 Transpose of rank-two tensors.

- a) Let A be a 4×4 -matrix. If we write the components of A as $A_{\mu\nu}$, $\mu, \nu = 0, \dots, 3$, how do we denote the components of A^T ?
- b) We usually denote the Lorentz matrix L in terms of its components $L^\mu{}_\nu$. How should we denote the components of the transpose matrix L^T ?

Problem 3 A very, very fast train.

A train moves with speed $v = 0.8c$. The train has length $L_0 = 100$ m at rest. The train approaches a straight tunnel with length $d_0 = 60$ m. Let us call the rest frame of the tunnel S , and the frame comoving with the train (i.e. the rest frame of the train) is called S' .

- a) Calculate the length of the train in the rest frame of the tunnel, S .
- b) Calculate the length of the tunnel in the rest frame of the train, S' .
- c) Is the train ever completely (with both ends) inside the tunnel? Explain your answer.

Problem 4 Inverse of L .

The four-vector $(x^\mu) = (ct, x, y, z)$ transforms under Lorentz transformations as $x'^\mu = L^\mu{}_\nu x^\nu$. In this problem you will find an expression for L^{-1} by looking at the example of boosts along the x -axis.

- a) Write down the matrix L for boosts along the x -axis.
- b) Calculate $L^T g L$. Where g is the matrix

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- c) Use the result from b) to find L^{-1} .