

FYS3120 Classical Mechanics and Electrodynamics

Discussion 6.2

March 3, 2025

Problem 1

- a) What is the metric tensor?
- b) What is tensor rank?
- c) What is a Lorentz invariant?
- d) Which of the following expressions makes sense using the tensor notation with spacetime indices?
 - (i) $A^\mu = B_{\mu\nu}A^\mu$
 - (ii) $C^\mu = T^\mu{}_{\nu\rho}A_\rho$
 - (iii) $C^\nu = T^\mu{}^\rho{}_\mu G^{\mu\nu}$
 - (iv) $D_\mu = \partial_\mu A_\nu$
- e) Let $f(x)$ and $g(x)$ be two scalar fields that are functions of spacetime. Find an alternative expression for the following, $\partial_\mu(f(x)g(x))$.
- f) What is the difference between ∂_μ and ∂^μ ?
- g) Consider a four-vector, D_μ with components $D_\mu = (f, \mathbf{D})$, and $\mathbf{D} = \mathbf{D}(\mathbf{r}, t)$ is a vector field in three-dimensional space, and $f(\mathbf{r}, t)$ is a scalar field. Write the following on covariant form:

$$\frac{1}{c} \frac{\partial f}{\partial t} - \nabla \cdot \mathbf{D}.$$

Problem 2 Transformation of rank-two tensors

- a) How does a tensor field, $F^{\mu\nu}$, transform under Lorentz transformations?
- b) What is the difference between $L^\mu{}_\nu$ and $L_\sigma{}^\rho$? Does it make sense to write L_ν^μ ?
- c) How does the metric $g_{\mu\nu}$ transform under Lorentz transformations in special relativity?
- d) Show that the full contraction $F^{\mu\nu}F_{\mu\nu}$ is a Lorentz scalar, where $F^{\mu\nu}$ is a (rank two) tensor field.
- e) Show that the inverse of $L^\mu{}_\nu$ can be written as $L_\sigma{}^\nu$. You may need the transformation property of the metric tensor.

Problem 3 A very, very fast train 2.¹

A train has length L_0 at rest. The train approaches a straight tunnel, with length at rest $d_0 < L_0$. Let us call the rest frame of the tunnel S , and the frame co-moving with the train is called S' . The train is moving at a very, very high velocity v in the positive x -direction in S , and the tunnel is located along the x -axis.

The length of the train in S gets length contracted such that $L' = \frac{1}{\gamma}L_0 = d_0$.

- a) Draw the Minkowski diagram where the tunnel is at rest. Draw the worldlines of the front and back end of the train in this Minkowski diagram.
- b) Draw the Minkowski diagram for the situation in S' .
- c) Explain if the train is fully inside the tunnel in using the diagrams you have made.

¹In case you want to do something other than notation drills today.