

FYS3120 Classical Mechanics and
Electrodynamics

Discussion 7

March 24, 2025

Problem 1

- a) What is an instantaneous inertial reference frame of a moving object?
- b) What is proper acceleration?
- c) Why (and under what conditions) does $E = mc^2$?
- d) What is the kinetic energy of an object moving with a relativistic velocity?
- e) What is hyperbolic motion?

Problem 2 Index notation

Index notation is a way to represent relations between vectors (and matrices) in terms of their indices (or components). For example, in a d -dimensional space we have the vectors $\mathbf{v} = (v_1, \dots, v_d)$, $\mathbf{u} = (u_1, \dots, u_d)$. If the sum of these two vectors equals the vector $\mathbf{w} = (w_1, \dots, w_d)$, then in vector notation we would write $\mathbf{u} + \mathbf{v} = \mathbf{w}$. Using index notation we can express the same relation as $u_i + v_i = w_i$. This is also often referred to as *component form* or *index form*.

- a) Use index notation to write $m\ddot{\mathbf{r}} = -\nabla V$. Here it might be a good idea to use $(x, y, z) = (x_1, x_2, x_3)$ for the components of $\mathbf{r}$.
- b) Consider an $n \times n$ matrix, A , with components a_{ij} , and a vector $\mathbf{x}$ with n components. Write the matrix-vector product $A\mathbf{x}$ using index notation.
- c) The **Levi-Civita symbol** is ϵ_{ijk} . Each of the indices i, j, k takes values 1, 2 and 3. We have that

$$\epsilon_{ijk} = \begin{cases} 1 & \text{if } ijk \text{ are cyclic} \\ -1 & \\ 0 & \text{otherwise} \end{cases}$$

We introduce matrices A, B and C . They have components $a_{ij} = \epsilon_{1ij}$, $b_{ij} = \epsilon_{2ij}$, $c_{ij} = \epsilon_{3ij}$, respectively. Write out the matrices A, B and C .

- d) Let $\mathbf{A} = A_x \mathbf{e}_x + A_y \mathbf{e}_y + A_z \mathbf{e}_z$ be a vector field where the components are smooth functions of x, y and z . Use index notation to show that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$.

Problem 3

- Group 1: Let $F^{\mu\nu}$ be rank-two tensor (it transforms according to the Lorentz transformations under boosts and rotations). Show that $F^{\mu\nu}F_{\mu\nu}$ is Lorentz invariant.
- Group 2: Let an arbitrary Lorentz transformation be denoted $L^\mu{}_\nu$. Show that the inverse of $L^\mu{}_\nu$ can be written as $L_\sigma{}^\nu$.
- Group 3: A particle follows the worldline $x^\mu(\tau)$ in spacetime, where τ is the particle's proper time. Show that $\frac{d^2x^\mu}{d\tau^2}$ is a space-like four-vector.