

**Written home exam in FYS3120****2021 V21****Duration: May 31 09:00 to May 31 13:00**

**It is important that you read this cover page carefully before you start.**

**General information:**

- Important messages during the exam are given directly from the course teacher on the course's semester page or in Canvas (if used in your course). It is therefore important that you check the course's semester page / Canvas room regularly.
- Your answer should reflect your own independent work and should be a result of your own learning and work effort.
- All sources of information are allowed for written home exams. If you reproduce a text from books, online articles, etc., a reference to these sources must be provided to avoid suspicions of plagiarism. This also applies if a text is translated from other languages.
- You are responsible for ensuring that your exam answers are not available to others during the exam period, neither physically nor digitally.
- Remember that your exam answers must be anonymous; do not state either your name or that of fellow students.
- If you want to withdraw from the exam, press the hamburger menu at the top right of Inspera and select "Withdraw".

**Collaboration during the exam:**

It is not allowed to collaborate or communicate with others during the exam. Cooperation and communication will be considered as attempted cheating. A plagiarism control is performed on all submitted exams where text similarities between answers are checked. If you use notes that have been prepared in collaboration with others before the exam, this might be detected in a plagiarism control. Such text similarities will be considered an attempt at cheating.

**Cheating:** Read about what is considered cheating on UiO's website.

**Digital file upload:**

You have been given 30 minutes extra time for uploading files (e.g. digital hand drawings). Check out how to submit digital hand drawings.

**Contact information:** User support exam.

This final exam accounts for roughly 80% of the total grade in FYS3120. The best possible score on this exam is 80 points. Up to four points will be given for clear, concise and well presented answers, including appropriate figures and/or diagrams. You may give your answers either in English or Norwegian. Good luck!

### Question 1 Statutory problem [22 points]

The Italian exchange student Michelangelo has absconded with the statue from the mechanics exam in 2019 in the back of his Fiat pick-up truck, see Fig. 1. He has cleverly engineered the back of the truck so that the statue is subject to a position dependent potential  $U$  to avoid the statue falling off. With our department head Professor Viefers in hot pursuit the upright statue starts sliding. Professor Viefers stops to ponder whether the problem is best considered in the Fiat reference frame  $F$  or in the reference frame  $S$  of a, presumably innocent, student bystander at rest with respect to the road. Please help her decide before Michelangelo gets away!



Figure 1: Statue from the FYS-MEK1110 V19 exam.

The path of the car in  $S$  is  $\mathbf{R}(t)$  with velocity  $\mathbf{V}(t)$ , which is a function of time alone. The coordinates of the statue are  $\mathbf{r}_0$  in  $S$  and  $\mathbf{r}$  in  $F$ . The Fiat is non-relativistic.

- a) In the student reference frame  $S$  the Lagrangian of the statue is

$$L(\mathbf{r}_0, \dot{\mathbf{r}}_0, t) = \frac{1}{2}m\dot{\mathbf{r}}_0^2 - U(\mathbf{r}_0 - \mathbf{R}). \quad (1)$$

Show that in  $F$  this Lagrangian can be written as

$$L(\mathbf{r}, \dot{\mathbf{r}}, t) = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\mathbf{V} \cdot \dot{\mathbf{r}} + \frac{1}{2}m\mathbf{V}^2 - U(\mathbf{r}). \quad (2)$$

[3 points]

- b) Argue why terms of the form  $W(t)$ , functions purely of time, can be removed from a Lagrangian without changing the equations of motion. [3 points]

- c) Show that the Lagrangian

$$L(\mathbf{r}, \dot{\mathbf{r}}, t) = \frac{1}{2}m\dot{\mathbf{r}}^2 - m\dot{\mathbf{V}} \cdot \mathbf{r} - U(\mathbf{r}), \quad (3)$$

has the same equations of motion as the Lagrangian in (2). [4 points]

- d) Find the equations of motion of the statue in  $F$  from Eq. (3). Give a brief interpretation of the physical meaning of the terms. [6 points]
- e) Given that the potential is  $U(\mathbf{r}) = \frac{1}{2}m\omega^2(x^2 + y^2)$ ,<sup>1</sup> where  $\mathbf{r} = (x, y, z)$ . Describe the movement of the statue in  $F$  if it starts at rest at  $\mathbf{r} = 0$  at the time  $t = 0$ , and the car experiences a constant acceleration  $a$  in the  $y$ -direction for some time, before the acceleration becomes zero again. [6 points]

## Question 2 Heffalon production [26 points]

You are being asked to design the new Very Large Hadron Collider (VLHC) at CERN which will collide beams of highly relativistic protons. In order to search for very massive new particles such as the heffalon,<sup>2</sup> the collider needs to have as large as possible *centre-of-mass energy*, meaning the total energy of the colliding protons in the centre-of-mass reference frame.

In the following, let us for concreteness name the relativistic momenta of the colliding particles  $\mathbf{p}_1$  and  $\mathbf{p}_2$ , their relativistic energies  $E_1$  and  $E_2$ , and their mass  $m_p$ .

- a) Define the centre-of-mass reference frame for the VLHC. [2 points]
- b) Explain why the maximum mass of new particles that can be produced at such a collider is given by  $M = E/c^2$ , where  $E$  is the centre-of-mass energy. [3 points]
- c) One design option for the VLHC is a *fixed target collider* where one proton is at rest in a target in the lab and the other proton is shot into it at high velocity. Write down the four-momenta for the two protons for this collider in the lab reference frame. [3 points]
- d) Find the centre-of-mass energy for the fixed target collider in terms of the incoming proton's energy  $E_1$  in the lab frame. [4 points]
- e) The other design option considered is a *symmetric collider* where the protons have the same magnitude of momentum in the lab frame, going in opposite directions. Find the centre-of-mass energy for this collider. [3 points]
- f) If the cost of making a collider to accelerate the protons to a given  $E_1$  or  $E_2$  in a single beam in the lab frame is linear in the energy, which design should you choose in order to reach the highest heffalon masses? [3 points]

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<sup>1</sup>What did you expect? These are physicists!

<sup>2</sup>See, e.g., A.J. Barr, C.G. Lester, *A search for direct heffalon production using the ATLAS and CMS experiments at the Large Hadron Collider*, arXiv:1303.7367 [hep-ph].

- g) In collider physics the centre-of-mass energy is often written in terms of the variable  $s$ , with  $s = (p_1 + p_2)^2 = (p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu})$  where  $p_1^\mu$  and  $p_2^\mu$  are the four-momenta of the colliding particles. Find the relationship between  $s$  and the centre-of-mass energy  $E$ . How does  $s$  change between reference frames? [3 points]
- h) Two other variables  $t$  and  $u$  are used when the collision produces two (and only two) outgoing particles (say Heffalons) with four-momenta  $p_3^\mu$  and  $p_4^\mu$ , defined by  $t = (p_1 - p_3)^2$  and  $u = (p_1 - p_4)^2$ . Show that

$$s + t + u = 2m_p^2 c^2 + 2m_h^2 c^2, \quad (4)$$

where  $m_h$  is the heffalon mass. [5 points]

### Question 3 Dipole dervish [28 points]

Two charges  $q_1 = -q$  and  $q_2 = q$  are placed at  $\mathbf{r}_1 = (-d/2, 0, 0)$  and  $\mathbf{r}_2 = (d/2, 0, 0)$ , respectively, and initially held fixed there.

- a) Use the multipole approximation for the scalar potential  $\phi(\mathbf{r})$  at large distances from the charges,  $r \gg d$ , and find the contributions up to and including terms proportional to  $1/r^3$ . [6 points]
- b) Find the resulting large distance approximation for the electric and magnetic fields. [3 points]

The charges are now rotated counterclockwise in the  $xy$ -plane in a circle of radius  $d$  around the  $z$ -axis with angular frequency  $\omega$ . We remind you that the scalar potential for time-dependent sources in the long distance approximation is given as

$$\phi(\mathbf{r}, t) = \frac{Q}{4\pi\epsilon_0 r} + \frac{\mathbf{r} \cdot \dot{\mathbf{p}}_{\text{ret}}}{4\pi\epsilon_0 r^2 c} + \frac{\mathbf{r} \cdot \mathbf{p}_{\text{ret}}}{4\pi\epsilon_0 r^3} + \dots, \quad (5)$$

where  $Q$  is the total charge and  $\mathbf{p}$  is the electric dipole moment, and where “ret” indicates that the quantity is evaluated at the retarded time  $t_r = t - r/c$ . The vector potential is similarly given as

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi r} \left( \dot{\mathbf{p}} + \frac{1}{c} \dot{\mathbf{m}} \times \hat{\mathbf{n}} + \frac{1}{2c} \ddot{\mathbf{D}}_{\hat{\mathbf{n}}} + \dots \right)_{\text{ret}}, \quad (6)$$

where  $\mathbf{m}$  is the magnetic dipole moment,  $\mathbf{D}_{\hat{\mathbf{n}}}$  is the electric quadrupole vector, and  $\hat{\mathbf{n}}$  is the unit vector in the  $\mathbf{r}$  direction.

- c) Find the electric dipole moment of the charges. [3 points]
- d) Find the magnetic dipole moment of the charges. [3 points]
- e) Find the electric quadrupole vector of the charges. [4 points]

- f) From the approximations in Eqs. (5) and (6), show that up to and including terms of order  $1/r^2$  the resulting electric field is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0 cr^2} [3\hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \dot{\mathbf{p}}) - \dot{\mathbf{p}}]_{\text{ret}} + \frac{\mu_0}{4\pi r} (\ddot{\mathbf{p}}_{\text{ret}} \times \hat{\mathbf{n}}) \times \hat{\mathbf{n}}. \quad (7)$$

*Hint:* The following product rule of differentiation may come in handy

$$\nabla(\mathbf{a} \cdot \mathbf{b}) = (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}). \quad (8)$$

This problem is the hardest problem in the exam, and you should only spend significant time on it if you have completed the rest. [5 points]

- g) Compare the results in **b** and **f**. Comment in particular on the physical interpretation of the terms in Eq. (7). [4 points]