

UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Analytical Mechanics and Electrodynamics (FYS3120)

Day of exam: March 20, 2026

Exam hours: 3 hours

This examination paper consists of 3 pages.
(including the title page)

Appendices: none

Permitted materials:

- approved calculator
- Rottman: *Matematisk formelsamling*
- 8 page document provided on semesterpage: *FYS3120 Analytical Mechanics and Electrodynamics. Collection of formulae*

Make sure that your copy of this examination paper is complete before answering !

Midterm exam

Spring 2026: Analytical Mechanics and Electrodynamics (FYS3120)

↪ *This mid-term exam accounts for roughly 20% of the total grade in FYS3120.
You may give your answers either in English or in Norwegian.
Maximal number of available points: 20.*

Good luck!

Problem 1 – Charged particle motion in a potential

A small body with mass m and charge q is moving in the horizontal plane (x - y plane), under the influence of a harmonic oscillator potential $V(r) = \frac{1}{2}m\omega_0^2 r^2$, and a constant magnetic field $\mathbf{B} = B\mathbf{k}$ which is directed perpendicular to the plane of the moving particle. The vector potential corresponding to $\mathbf{B}$ can be written as $\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B}$, with $\mathbf{r}$ as the position vector of the particle.

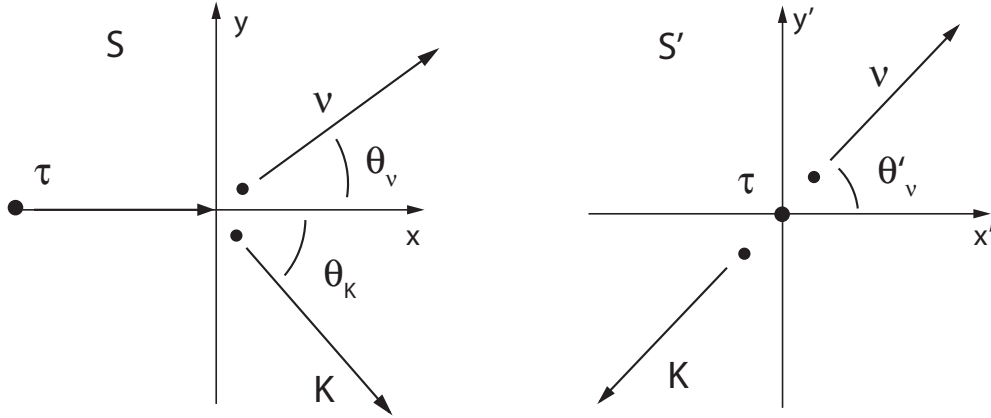
- a) With the polar coordinates of the plane (r, ϕ) used as generalized coordinates, show that the Lagrangian takes the form

$$L = \frac{1}{2}m \left(\dot{r}^2 + Ar^2(\dot{\phi}^2 + \omega\dot{\phi} - \omega_0^2) \right) \quad (1)$$

where A and ω are constants. Determine these constants!

- b) The polar angle ϕ is cyclic. Explain what that means and give the expression for the corresponding conserved quantity, which we label l . What is the physical interpretation of this quantity? The form of the Lagrangian implies that there is a second constant of motion. Give the expression and physical interpretation of that quantity.
- c) Establish Lagrange's equation of motion for the radial variable r , and use the cyclic property of ϕ to express the equation in the variable r alone.
- d) Show that the radial equation has solutions which describe circular motion, and provide the corresponding radius and angular velocity as functions of the parameters of the problem.
- e) Show that the radial equation also has a solution where the particle performs oscillations about the origin, in a direction which rotates with time, and find the oscillation and rotation frequencies. Finally, give a qualitative description of the general, more generic type of motion described by the Euler-Lagrange equations.

Problem 2 – particle decay



We consider a process where a negatively charged tau-lepton (τ^-) decays into a K -meson and a neutrino ν . The mass of the tau-lepton is $m_\tau = 1777 \text{ MeV}/c^2$, the mass of the K -meson is $m_K = 494 \text{ MeV}/c^2$, and the neutrino we consider as massless. In the figure the decay process is shown both in the laboratory frame S , and in the rest frame S' of the tau-lepton. The relative velocity of the two reference frames is $v = 0.9c$, with direction along the x -axis (in both frames).

- We study first the process in the rest frame S' . Use the relativistic conservation laws for energy and momentum to determine the energies E'_ν and E'_K , of the neutrino and the K -meson, respectively. Determine also the absolute value of the (three-) momenta of the two particles in this reference frame.
- Assume the direction of the neutrino in reference frame S' to be orthogonal to the relative velocity of the two frames, $\theta'_\nu = \pi/2$. Write the transformation formula for relativistic 4-momenta, and use it to determine the energies E_ν and E_K of the two particles in the lab frame S . Determine also the angles θ_ν and θ_K of the two particles.
- With no knowledge of the direction of the neutrino, explain why the probability is 50% for the neutrino to have an energy larger than the value found in b), and also 50% probability to find it within a smaller angle θ_ν than the value evaluated in b).