

FYS3120 Classical Mechanics and Electrodynamics

Problem set 1

Deadline for handing in in canvas: **3 February**, 3pm

Problem 1 Figure 1 shows four different mechanical systems, in **a)** a pendulum attached to a block which in turn is attached to a spring, **b)** a pendulum which is attached to a vertical ring which rotates with a fixed frequency ω , **c)** a straight rod which can tilt without sliding on the top of cylinder, while the cylinder can roll on a horizontal plane (no slipping), and **d)** a spinning top which moves on a horizontal floor.

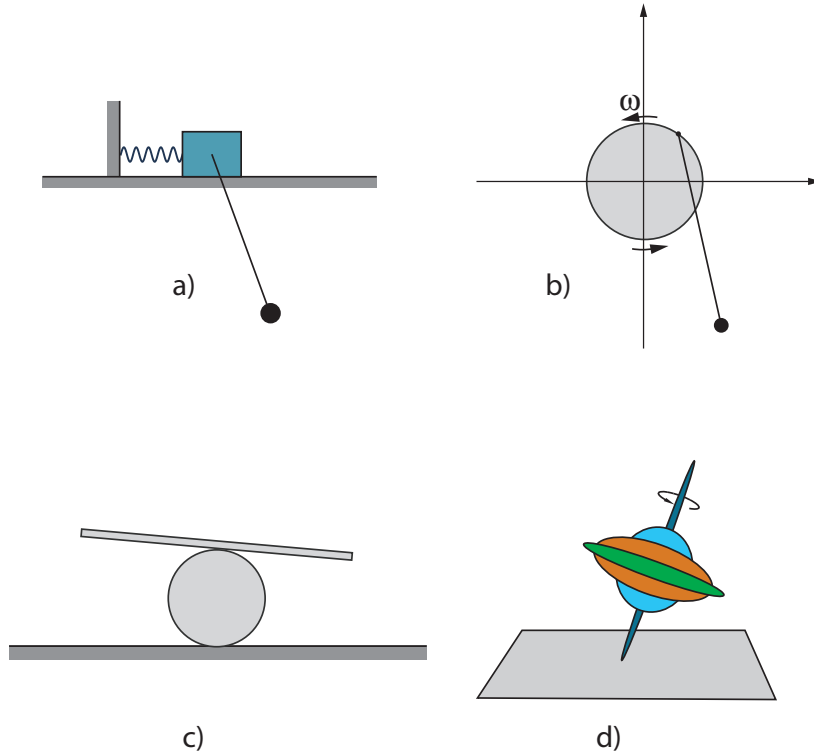


Figure 1: Four mechanical systems.

In all cases specify the number of degrees of freedom and choose an appropriate set of generalised coordinates.

Problem 2 A particle with mass m moves in three-dimensional space under the influence of a constraint. The constraint is expressed by the equation

$$e^{-(x^2+y^2)} + z = 0, \quad (1)$$

for the Cartesian coordinates (x, y, z) of the particle.

- a)** Explain why the number of degrees of freedom of the particle is two. Use x and y as generalised coordinates and find the expression for the position vector $\mathbf{r}$ of the particle in terms of x and y .

- b) A virtual displacement is a change in the position of the particle $\mathbf{r} \rightarrow \mathbf{r}' = \mathbf{r} + \delta\mathbf{r}$ which is caused by an infinitesimal change in the generalised coordinates, $x \rightarrow x' = x + \delta x$ and $y \rightarrow y' = y + \delta y$. Find $\delta\mathbf{r}$ expressed in terms of δx and δy .
- c) The constraint can be interpreted as a restriction for the particle to move on a two-dimensional surface in three-dimensional space. Any virtual displacement $\delta\mathbf{r}$ is a tangent vector to the surface while the constraint force $\mathbf{f}$ which acts on the particle is perpendicular to the surface. Use this to determine $\mathbf{f}$ (up to a normalization factor) as a function of x and y .
- d) Make a drawing of a section through the surface for $y = 0$. Indicate in the drawing the direction of the two vectors $\mathbf{f}$ and $\delta\mathbf{r}$ for a chosen point on the surface.

Problem 3 A pendulum consists of a rigid rod, which we consider as massless, and a pendulum bob of mass m . The point of suspension of the pendulum has horizontal coordinate $x = s$ and vertical coordinate $y = 0$. This set-up is shown in Fig. 2.

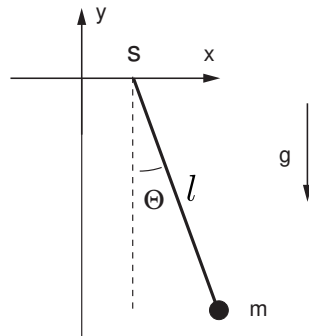


Figure 2: The pendulum problem.

- a) Assume first that the point of suspension is kept fixed, with $s = 0$. Use the angle θ as generalised coordinate, find the Lagrangian of the system and determine the form of Lagrange's equation for the system. Check that it has the standard form of a pendulum equation.
- b) Find and categorise any equilibrium points for the system.
- c) The point of suspension is now released so it can move freely in the horizontal direction (x -direction). Use s and θ as generalised coordinates for the system and determine the corresponding set of Lagrange's equations.

- d)** Show that s can be eliminated to give an equation which only depends on θ and its derivatives. Further show that this equation implies that the vertical motion of the pendulum bob is identical to free fall in the gravitational field (in reality restricted by the length l of the rod). *Warning:* Do not attempt to substitute directly into the Lagrangian. Try to think why this will give the wrong answer.