

FYS3120 Analytical Mechanics and
Electrodynamics

Problem set 11

Deadline for handing in (in canvas): **5 May**, 3pm

Problem 1 Exam 2009

A so-called **half-wave center-fed** antenna is formed by a thin linear conductor of length a . It is oriented along the z -axis as shown in Fig. 1. An alternating current is running in the antenna, of the form

$$I(z, t) = I_0 \cos \frac{\pi z}{a} \cos \omega t, \quad -a/2 < z < a/2. \quad (1)$$

In the following $\lambda(z, t)$ denotes the linear charge density of the antenna (charge per unit length). At time $t = 0$ the antenna is charge neutral, so that $\lambda(z, 0) = 0$.

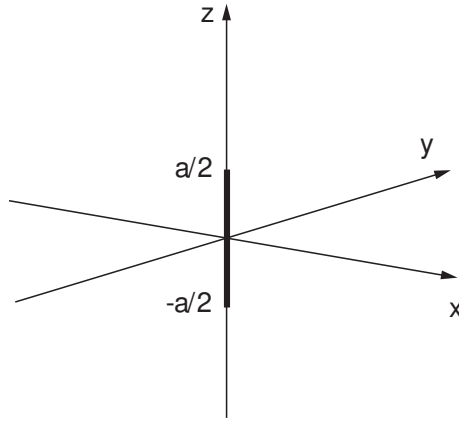


Figure 1: A half-wave centre-fed antenna of length a and centre at the origin.

- a) Show that the charge density and current satisfy the relation

$$\frac{\partial \lambda}{\partial t} + \frac{\partial I}{\partial z} = 0, \quad (2)$$

and find λ as a function of z and t .

- b) Show that the electric dipole moment of the antenna has the form

$$\mathbf{p}(t) = p_0 \sin \omega t \mathbf{e}_z, \quad (3)$$

with $\mathbf{e}_z$ as the unit vector along the z -axis, and determine the constant p_0 .

- c) Use the expressions for electric dipole radiation to determine the electric and magnetic fields in a point at a large distance r from the antenna on the x -axis. What is the type of polarization of the radiation from the antenna in this direction?

As a reminder, the general expressions for the radiation fields of an electric dipole positioned at the origin are

$$\mathbf{B}(\mathbf{r}, t) = -\frac{\mu_0}{4\pi c} \frac{\mathbf{n}}{r} \times \ddot{\mathbf{p}}_{ret}, \quad \mathbf{E}(\mathbf{r}, t) = c\mathbf{B}(\mathbf{r}, t) \times \mathbf{n} \quad (4)$$

with $\mathbf{p}_{ret} = \mathbf{p}(t - r/c)$ and $\mathbf{n} = \mathbf{r}/r$.

Problem 2 Exam 2006

A thin rigid rod of length ℓ rotates in the (x, y) -plane as shown in Fig. 2. At the two end points there are fixed charges of opposite sign, $+q$ and $-q$. The rod is rotating with constant angular frequency ω . This gives rise to a time dependent electric dipole moment

$$\mathbf{p}(t) = q\ell(\cos\omega t \mathbf{e}_x + \sin\omega t \mathbf{e}_y). \quad (5)$$

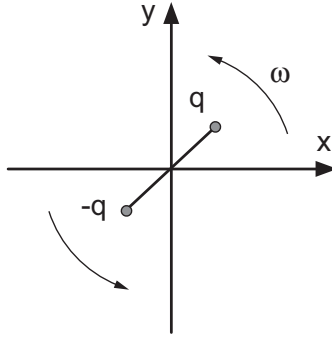


Figure 2: Rotating rod with charges.

- a) Use the general expression for the radiation fields of an electric dipole (see the formula collection of the course) to show that the magnetic field in the present case can be written as

$$\begin{aligned} \mathbf{B}(\mathbf{r}, t) = & B_0(r)(\cos\theta \sin(\omega(t - r/c)) \mathbf{e}_x - \cos\theta \cos(\omega(t - r/c)) \mathbf{e}_y \\ & - \sin\theta \sin(\omega(t - r/c) - \phi) \mathbf{e}_z \end{aligned} \quad (6)$$

with (r, θ, ϕ) as the polar coordinates of $\mathbf{r}$. Find the expression for $B_0(r)$.

What is the general relation between the electric field $\mathbf{E}(\mathbf{r}, t)$ and the magnetic field $\mathbf{B}(\mathbf{r}, t)$ in the radiation zone? (A detailed expression for $\mathbf{E}(\mathbf{r}, t)$ is not needed.)

- b) Show that radiation in the x -direction is linearly polarized. What is the polarization of the radiation in the z -direction?

- c) Find the time-averaged expression for the energy density of the radiation. In what direction has the radiated energy its maximum?