Home exam FYS3140 V2011

Problem1

a) Use the Fröbenius method (series expansion) to obtain a solution $y_1(x)$ of the differential equation

$$x(x-1)y''+(3x-1)y'+y=0.$$

(You should get $y_1(x)=\frac{1}{1-x}$).

b) Find a second solution $y_2(x)$ (Hint: variation of the constant).

Write down the general solution.

Discuss the regions of validity for your solution.

Problem 2

In this problem you shall use the Greens function method to find a solution of the differential equation

$$y''(t)-a^2y(t)=f(t),$$

where a is a real constant, and $f(t)$ is an unspecified function of t.

We want a solution for the range $t\in[0,\infty>$.

The initial conditions are $y(0)=y'(0)=0$.

a) Verify that the initial conditions for $y(t)$ are consistent with the conditions $G(0,t')=G'(0,t')=0$ for the Greens function $G(t,t')$.

Show that the solution is

$$y(t)=\int_0^t \frac{1}{a}\sinh a(t-t')f(t')dt'.$$

b) Assume that $f(t)$ represents a finite "shock" at $t=t_0>0$, described by $f(t)=k\delta(t-t_0)$, where k and t_0 are constants.

Determine $y(t)$ in this case.

c) Now, let $f(t)$ represent a "square pulse":

$$f(t)=\begin{cases} K, & t_1<t<t_2,\ t_1>0 \\ 0 & \text{otherwise} \end{cases},$$

where K, t_1 and t_2 are constants.

Determine $y(t)$ for this choice of $f(t)$. Discuss your solution for different regions of $t-$ values.

Problem 3

You shall in this problem use the Fourier transform to solve the differential equation

$$y''(t) - k^2 y(t) = e^{-a|t|}, \quad a \text{ and } k \text{ are a real constants, } a > 0.$$

a) Show that the Fourier transform $Y(\omega)$ of the solution $y(t)$ is given by

$$Y(\omega) = -\frac{2a}{\sqrt{2\pi}} \frac{1}{(\omega^2 + a^2)(\omega^2 + k^2)}.$$

b) Determine $y(t)$ for $k^2 \neq a^2$ (hint: complex integration).

What conditions do your solution fulfill? Is your solution continuous at $t = 0$?

c) Determine $y(t)$ for $k^2 = a^2$.

The Fourier transform: $F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$, $f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$.

Problem 4

The complex Fourier series is given by

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/L}, \quad c_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{-in\pi x/L} dx, \quad x \in [-L, L].$$

a) Show that

$$e^x = \frac{\sinh \pi}{\pi} \sum_{n=-\infty}^{\infty} (-1)^n \frac{1 + in}{1 + n^2} e^{inx}, \quad x \in [-\pi, \pi].$$

b) Determine the value of the sum

$$S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{1 + n^2}.$$

c) Use the series from a) to obtain the Fourier series for e^x in terms of $\cos nx$ and $\sin nx$.

Check the sum in b) from this series.

Due Monday April 4th at 14.00. To be turned in at the department office.

Write your candidate number on your paper, no name!