FYS3140. Second set of obligatory problems 2011

Problem 1

Find the inverse of the following Laplace transforms (the textbook uses p instead of s):

a) $\dfrac{2}{s^3(s+2)}$.

b) $\dfrac{s}{(s^2+a^2)(s^2+b^2)}$.

(Hint: Convolution theorem)

Problem 2

Use the Laplace transform to obtain the solutions $x(t)$ and $y(t)$ for the following set of coupled equations:

$$\ddot{x}(t)+2n\dot{x}(t)+n^2x(t)=0$$

$$\ddot{y}(t)+2n\dot{y}(t)+n^2y(t)=\mu\dot{x}(t).$$

Initial conditions: $x(0)=y(0)=\dot{y}(0)=0,\ \dot{x}(0)=\lambda$ (n,μ and λ are real constants).

Problem 3

Write down and solve the Euler equations that make the following integrals stationary:

a) $I=\int_{x_1}^{x_2} x\sqrt{1+y'^2}\,dx$.

(Answer: $ax=\cosh(ay+b)$, a and b are arbitrary constants).

b) $I=\int_{x_1}^{x_2}(1+yy')^2\,dx$.

(Hint: Show that the Euler equation is $y'^2+yy''=0$.

Answer: $x=ay^2+b$, a and b are arbitrary constants).

Problem 4

You shall in this problem obtain a solution of the two dimensional Laplace equation

$$\frac{\partial^2u(x,y)}{\partial x^2}+\frac{\partial^2u(x,y)}{\partial y^2}=0$$

for the region $0\le x\le a$ and $0\le y\le b$.

a) Show that there is a solution of the form

$$u_n(x,y)=A_n\sin\frac{n\pi x}{a}\sinh\frac{n\pi y}{a},$$

with the boundary conditions u(x,0)=u(0,y)=u(a,y)=0.
Specify the possible values of n.

b) Find a solution which in addition to the boundary conditions given in a) also satisfies the conditions

$$u(x,b)=\begin{cases} x & 0\le x<a \\ 0 & x=a\ . \end{cases}$$

Problem 5

Given the partial differential equation

$$y\frac{\partial^2 u(x,y)}{\partial x^2}=\frac{\partial u(x,y)}{\partial y}\ ,$$

with the condition u(x,0)=δ(x).

We will try to solve this equation by use of a Fourier transform, with x as a variable and y as a parameter.

a) Find the Fourier transform F(ω,y) of u(x,y), and write down the integral that determines u(x,y).

b) Find u(x,y), and check u(x,0).

(You might need the integral $\int_{-\infty}^{\infty} e^{-\alpha(x+i\beta)^2}dx=\sqrt{\frac{\pi}{\alpha}}$, α and β are real numbers, $\alpha>0$.

See also Rottmann page 106 for $\delta(x)$)

Due Monday May 23rd at 14.00.

Write your name on your paper, no candidate number!