Format for delivery of report and programs

The format of the project is that of a printed file or hand-written report. The programs should also be included with the report. Write only your candidate number on the first page of the report and state clearly that this is your report for project 5 of FYS3150, fall 2011. There will be a box marked 'FYS3150' at the reception of the Department of Physics (room FV128).

Project 5, The solar system, deadline December 12, 3pm

We study first a hypothetical solar system with one planet, say Earth, which orbits around the Sun. The only force in the problem is gravity. Newton's law of gravitation is given by a force F_G

$$F_G = \frac{GM_{\mathrm{sun}}M_{\mathrm{Earth}}}{r^2},$$

where M_{sun} is the mass of the Sun and M_{Earth} is the mass of Earth. The gravitational constant is G and r is the distance between Earth and the Sun. We assume that the sun has a mass which is much larger than that of Earth. We can therefore safely neglect the motion of the sun in this problem. In the first part of this project, your aim is to compute the motion of the Earth using different methods for solving ordinary differential equations.

We assume that the orbit of Earth around the Sun is co-planar, and we take this to be the xy-plane. Using Newton's second law of motion we get the following equations

$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_{\mathrm{Earth}}},$$

and

$$\frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_{\mathrm{Earth}}},$$

where $F_{G,x}$ and $F_{G,y}$ are the x and y components of the gravitational force.

a) Rewrite the above second-order ordinary differential equations as a set of coupled first order differential equations. Write also these equations in terms of dimensionless variables. As an alternative to the usage of dimensionless variables, you could also use so-called astronomical units (AU as abbreviation). If you choose the latter set of units, one astronomical unit of length, known as 1 AU, is the average distance between the Sun and Earth, that is 1 AU = 1.5×10^{11} m. It can also be convenient to use years instead of seconds since years match better the solar system. The mass of the Sun is $M_{\mathrm{sun}} = M_{\odot} = 2\times10^{30}$ kg. The mass of Earth is $M_{\mathrm{Earth}} = 6\times10^{24}$ kg. The mass of other planets like Jupiter is $M_{\mathrm{Jupiter}} = 1.9\times10^{27}$ kg and its distance to the Sun is 5.20 AU. Similar numbers for Mars are $M_{\mathrm{Mars}} = 6.6\times10^{23}$ kg and 1.52 AU, for Venus $M_{\mathrm{Venus}} = 4.9\times10^{24}$ kg and 0.72 AU, for Saturn are $M_{\mathrm{Saturn}} = 5.5\times10^{26}$ kg and 9.54 AU, for Mercury are $M_{\mathrm{Mercury}} = 2.4\times10^{23}$ kg and 0.39 AU, for Uranus are $M_{\mathrm{Uranus}} = 8.8\times10^{25}$ kg and 19.19 AU, for Neptun are $M_{\mathrm{Neptun}} = 1.03\times10^{26}$ kg and 30.06 AU and for Pluto are $M_{\mathrm{Pluto}} = 1.31\times10^{22}$ kg and 39.53 AU. Pluto is no longer considered a planet, but we add it here for historical reasons.

   Finally, mass units can be obtained by using the fact that Earth's orbit is almost circular around the Sun. For circular motion we know that the force must obey the following relation

$$F_G = \frac{M_{\mathrm{Earth}}v^2}{r} = \frac{GM_{\odot}M_{\mathrm{Earth}}}{r^2},$$

   where v is the velocity of Earth. The latter equation can be used to show that

$$v^2r = GM_{\odot} = 4\pi^2\mathrm{AU}^3/\mathrm{yr}^2.$$

   Discretize the above differential equations and set up an algorithm for solving these equations using the so-called Euler-Cromer method discussed in the lecture notes, chapter 8.

b) Write then a program which solves the above differential equations for the Earth-Sun system using the Euler-Cromer method. Find out which initial value for the velocity that gives a circular orbit and test the stability of your algorithm as function of different time steps Δt. Find a possible maximum value Δt for which the Euler-Cromer method does not yield stable results. Make a plot of the results you obtain for the position of Earth (plot the x and y values) orbiting the Sun.

   Check also for the case of a circular orbit that both the kinetic and the potential energies are constants. Check also that the angular momentum is a constant. Explain why these quantities are conserved.

c) Modify your code by implementing the fourth-order Runge-Kutta method and compare the stability of your results by repeating the steps in b). Compare the stability of the two methods, in particular as functions of the needed step length Δt. Comment your results.

d) Kepler's second law states that the line joining a planet to the Sun sweeps out equal areas in equal times. Modify your code so that you can verify Kepler's second law for the case of an elliptical orbit. Compare both the Runge-Kutta method and the Euler-Cromer method and check that the total energy and angular momentum are conserved. Why are these quantities conserved? A convenient choice of starting values are an initial position of 1 AU and an initial velocity of 5 AU/yr.

e) Consider then a planet which begins at a distance of 1 AU from the sun. Find out by trial and error what the initial velocity must be in order for the planet to escape from the sun. Can you find an exact answer?

f) We will now study the three-body problem, still with the Sun kept fixed at the center but including Jupiter (the most massive planet in the solar system, having a mass that is approximately 1000 times smaller than that of the Sun) together with Earth. This leads us to a three-body problem. Without Jupiter, Earth's motion is stable and unchanging with time. The aim here is to find out how much Jupiter alters Earth's motion.

   The program you have developed can easily be modified by simply adding the magnitude of the force betweem Earth and Jupiter.

   This force is given again by

$$F_{\text{Earth}-\text{Jupiter}} = \frac{GM_{\text{Jupiter}}M_{\text{Earth}}}{r^2_{\text{Earth}-\text{Jupiter}}},$$

   where M_{Jupiter} is the mass of the sun and M_{Earth} is the mass of Earth. The gravitational constant is G and $r_{\text{Earth}-\text{Jupiter}}$ is the distance between Earth and Jupiter.

   We assume again that the orbits of the two planets are co-planar, and we take this to be the xy-plane. Modify your first-order differential equations in order to accomodate both the motion of Earth and Jupiter by taking into account the distance in x and y between Earth and Jupiter. Set up the algorithm and plot the positions of Earth and Jupiter using the fourth-order Runge-Kutta method.

   As you will notice, the influence on Earth from Jupiter is very small. Repeat these calculations by increasing the mass of Jupiter by a factor of 10 and 1000 and plot the position of Earth.

g) Finally, we carry out a real three-body calculation where all three systems, Earth, Jupiter and the Sun are in motion. To do this, choose the center-of-mass position of the three-body system as the origin rather than the position of the sun. Give the sun an initial velocity which makes the total momentum of the system exactly zero (the center-of-mass will remain fixed). Compare these results with those from the previous exercise and comment your results.

h) This part is optional but gives you an additional 30% on the final score! Extend your program to include all planets in the solar system (do not include the various moons) and discuss your results. Try to find data for the intial positions and velocities for all planets.