

UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

FYS3220 – Lineær kretselektronikk

Digital signalbehandling

Analog til digital

Sverre Holm

Analog til digital-omforming

- Kvantisering
  - Kvantiseringsstøy: SNR [db] $\propto$ 6 B [bits]
  - Overstyring
  - Dithering: Ved periodisk signal og ved lavt nivå
- Samplingsteoremet
  - Utledning ved analog/digital Fourier
  - Aliasing
  - Samplingsskop og båndpassampling

ADC

- Analog

Deler digitalisering i to:

1. Sampling
2. Kvantisering

- Kvantiseringsfeil

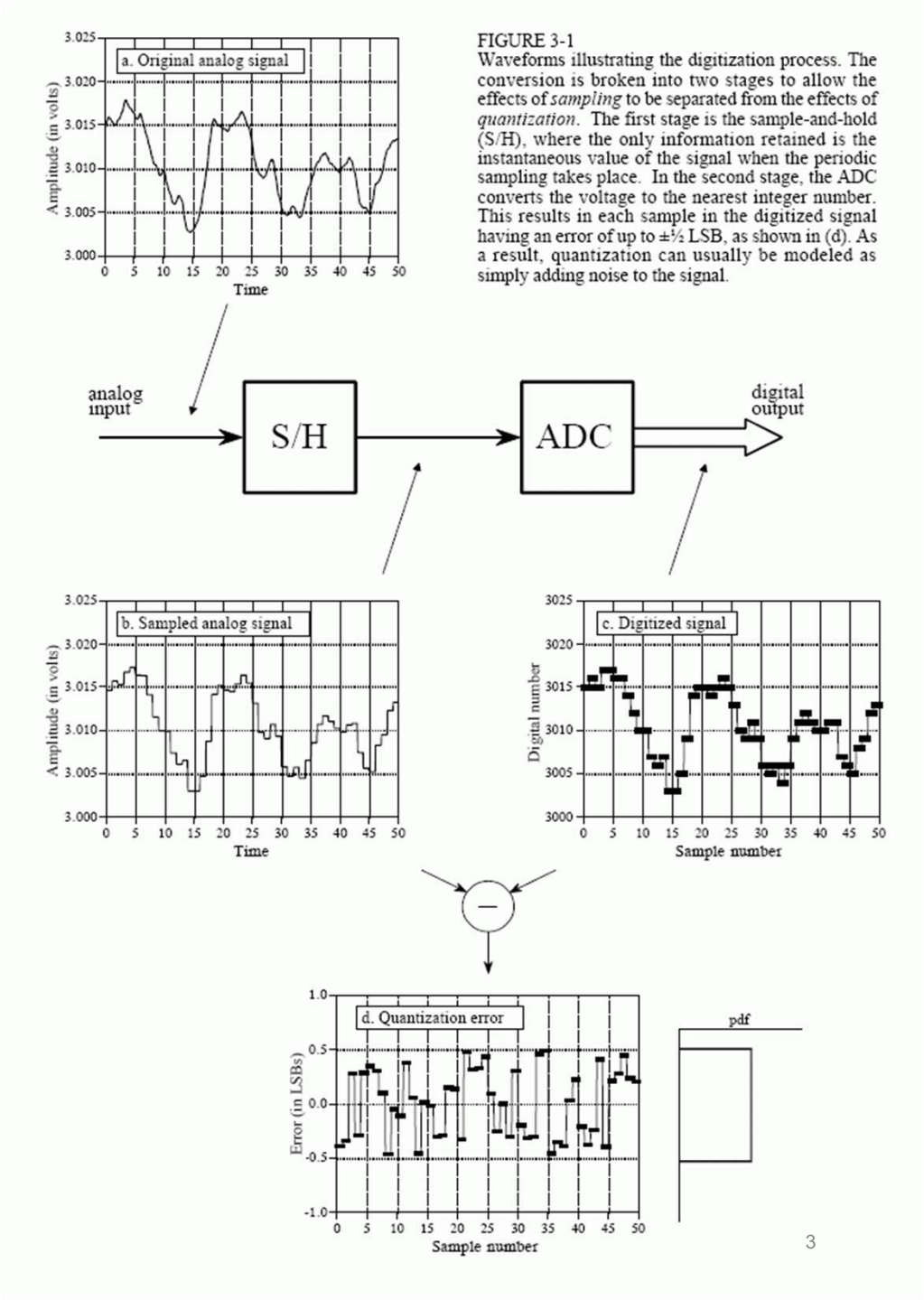


FIGURE 3-1
Waveforms illustrating the digitization process. The conversion is broken into two stages to allow the effects of *sampling* to be separated from the effects of *quantization*. The first stage is the sample-and-hold (S/H), where the only information retained is the instantaneous value of the signal when the periodic sampling takes place. In the second stage, the ADC converts the voltage to the nearest integer number. This results in each sample in the digitized signal having an error of up to ±½ LSB, as shown in (d). As a result, quantization can usually be modeled as simply adding noise to the signal.

Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

Kvantisering

- Uniform kvantisering: trinnene er like
  - B bits $\Rightarrow$ L=2^B trinn; I praksis vanlig med B=8, 12, 16, 24
- Avrunding, avkorting (trunkering), fortegns-avkorting

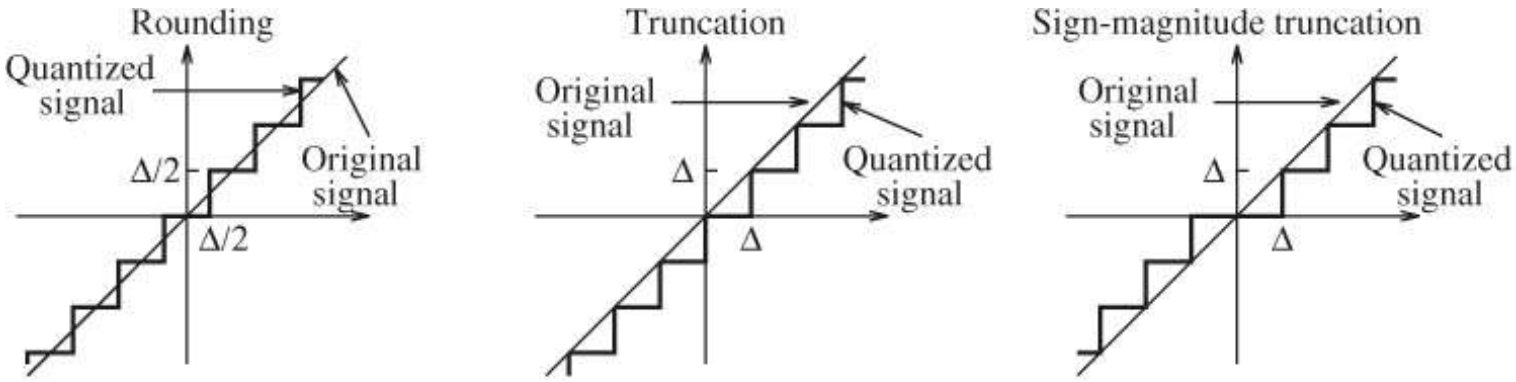
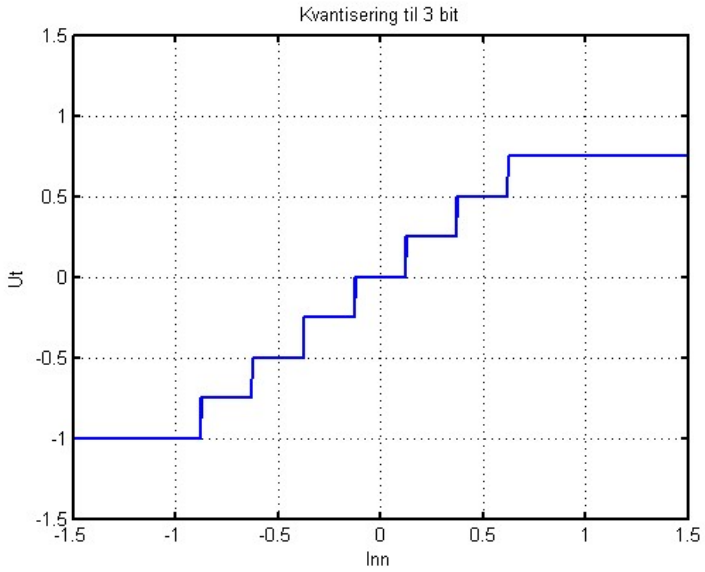


FIGURE 7.22 Various ways of quantizing a signal. The quantized value is chosen as the nearest quantization level when rounding or the next lower level when truncating. In sign-magnitude truncation, the absolute value (magnitude) is truncated and the actual sign is restored afterwards

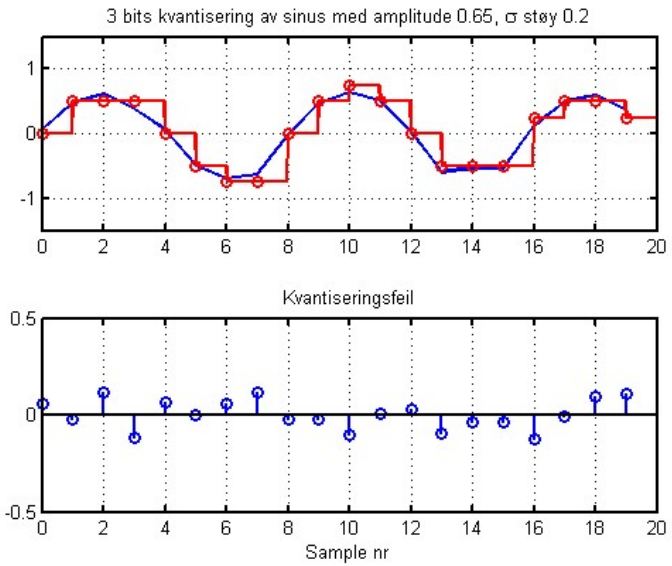
3-bits kvantiserer (avrunding)



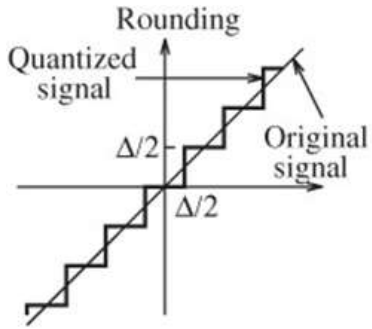
- Amplitudeområde : +/- 1
- Antall trinn: $2^3=8$
- Hvert trinn er:

  $2/2^3 = 0.25$

3-bits kvantisering av sinus i støy: kvantiseringsfeil ~ støy



- Hvert trinn er 0.25
- Maks feil er …



Kvantisering

- Hva skjer hvis inngangsverdi er over maks-verdien?
- Klipping
  - Metning eller nulling

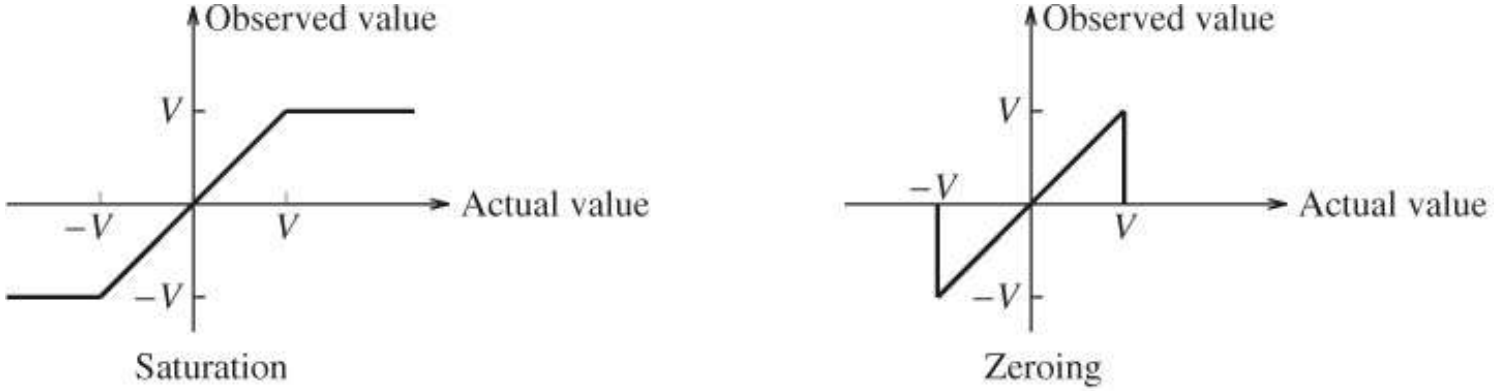
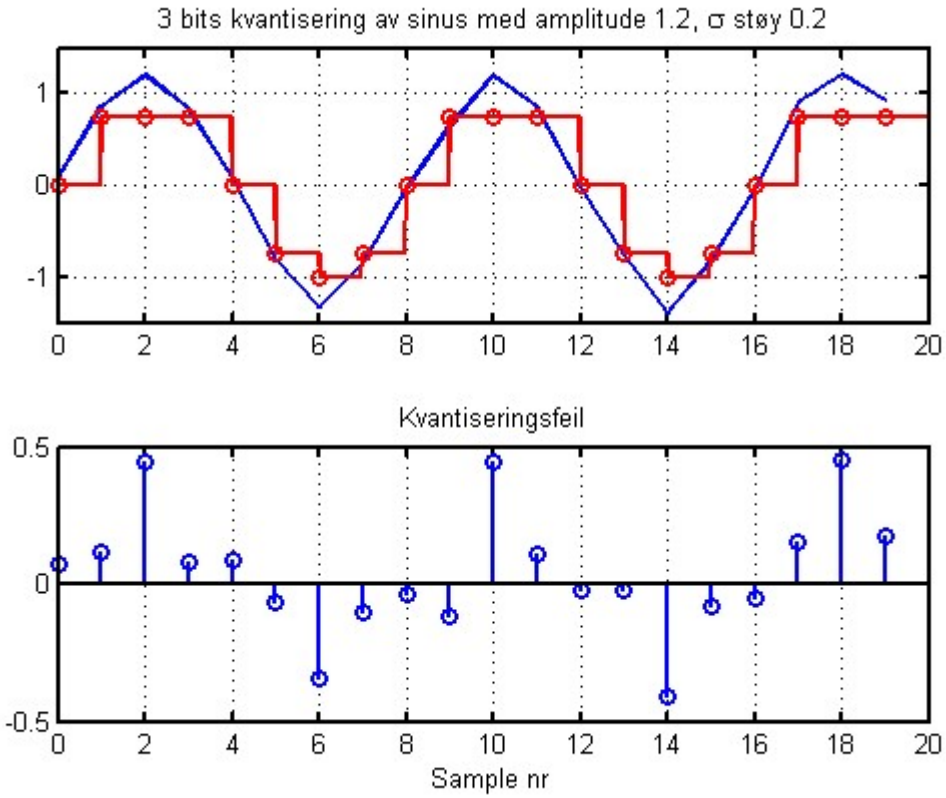


FIGURE 7.23 Two overflow characteristics. In saturation, values outside the full-scale range are set to the full-scale value itself. In zeroing, values outside the full-scale range are set to zero

- Demo: Kvantisering.m → leder opp til figur neste side

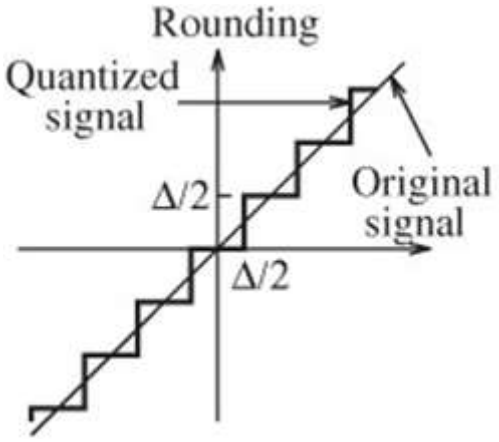
Overstyring: sinus i støy ved metning



Kvantiseringfeil - kvantiseringstøy

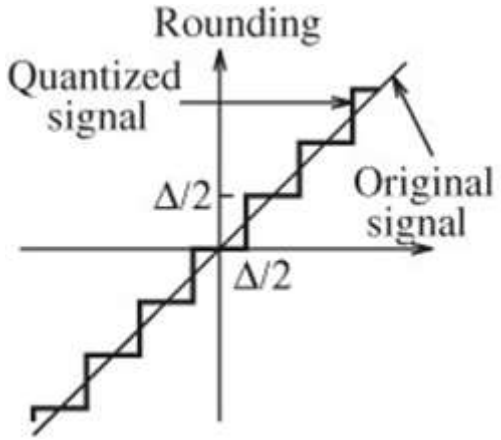
- Feil $\varepsilon = x[n] - x_Q[n]$
- Maks dynamikk $D = x_{max} - x_{min}$
- Kvantiseringstrinn $\Delta = D/L = D/2^B$
- Dynamikk $\approx D/\Delta = 2^B$
  - dB: $20\log(2^B) = 20\log 2 \cdot B = 6 \cdot B$
  - NB! Øker med 6 dB pr bit
  - Mer nøyaktig beregning følger!

- Hvilke forenklinger er gjort her?

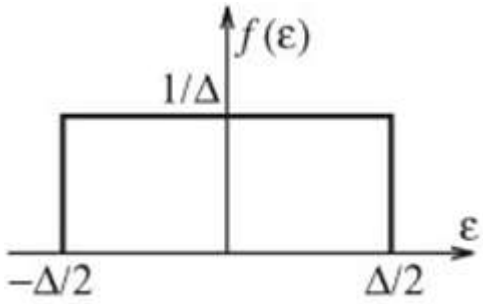


Kvantiseringfeil - kvantiseringstøy

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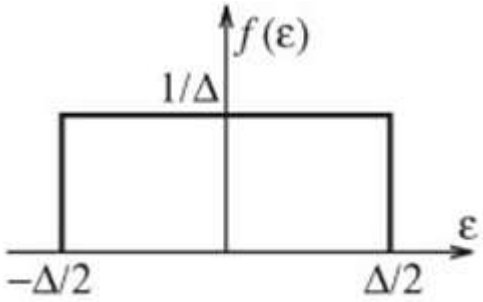


- Feil, statistisk: $-\Delta/2 < \varepsilon < \Delta/2$
- B stor nok $\Rightarrow$ feil uniformt fordelt, & ukorrellert med signal



Kvantiseringsfeilen

- Effekt i kvantiseringsstøyen
- pdf: f(ε) = 1/Δ for |ε| < Δ/2
- Støyeffekt = variansen:



$$P_N = \sigma^2 = \int_{-\infty}^{-\infty} \epsilon^2 f(\epsilon) d\epsilon = \frac{1}{\Delta} \int_{-\Delta/2}^{-\Delta/2} \epsilon^2 d\epsilon = \frac{\Delta^2}{12}$$

- Trinn: Δ = D/L = dynamikk / antall nivåer
- Effekt i støyen: $10\log(P_N)=10\log(D^2/12L^2)=20\log D-20\log L-10.8$

Signal-støy forhold, B-bits kvantisering

- Signal til støy forhold: SNR=P_S/P_N:

$$SNR = 10\log(P_S/P_N) = 10\log(P_S) + 20\log(L) - 20\log(D) + 10.8$$

- Uttrykt ved antall bit, 20log(L)=20log(2^B)=6B:

$$SNR = 10\log(P_S/P_N) = 10\log(P_S) + 6B - 20\log(D) + 10.8$$

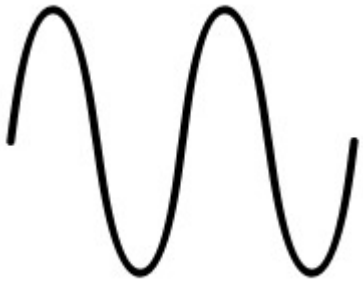
  - SNR $\propto$ 6B: 6 dB økning i SNR og dynamikk pr bit
  - Hvis dynamikk, D, er for stor $\Rightarrow$ SNR faller, dvs hvis maks nivå sjelden overskrides, faller SNR
  - Avveining mellom mer kvantiseringsstøy eller flere feil pga metning av A/D-omformer
  - Skal bruke denne formelen videre

Kvantiseringseffekter, eksempel

- Nivå mellom +/-2V, Hvor mange bit trengs for å få < 5mV rms kvantiseringsfeil?
  - D = 4 V, rms = $\sigma = 5$ mV
  - $\sigma = \Delta/12^{0.5} \Rightarrow \Delta = 12^{0.5} \cdot \sigma$ = 17.3 mV
  - og $\Delta = D/2^B \Rightarrow 2^B = D/\Delta$ = 4000 mV/17.3 mV = 231
  - $B = \log_2(231) \approx 7.85$, dvs B=8 bit
    - Sjekk at da blir $\sigma = 4.5$ mV

Eks: Kvantisering av sinus

$$SNR = 10\log(P_S) + 6B - 20\log(D) + 10.8$$



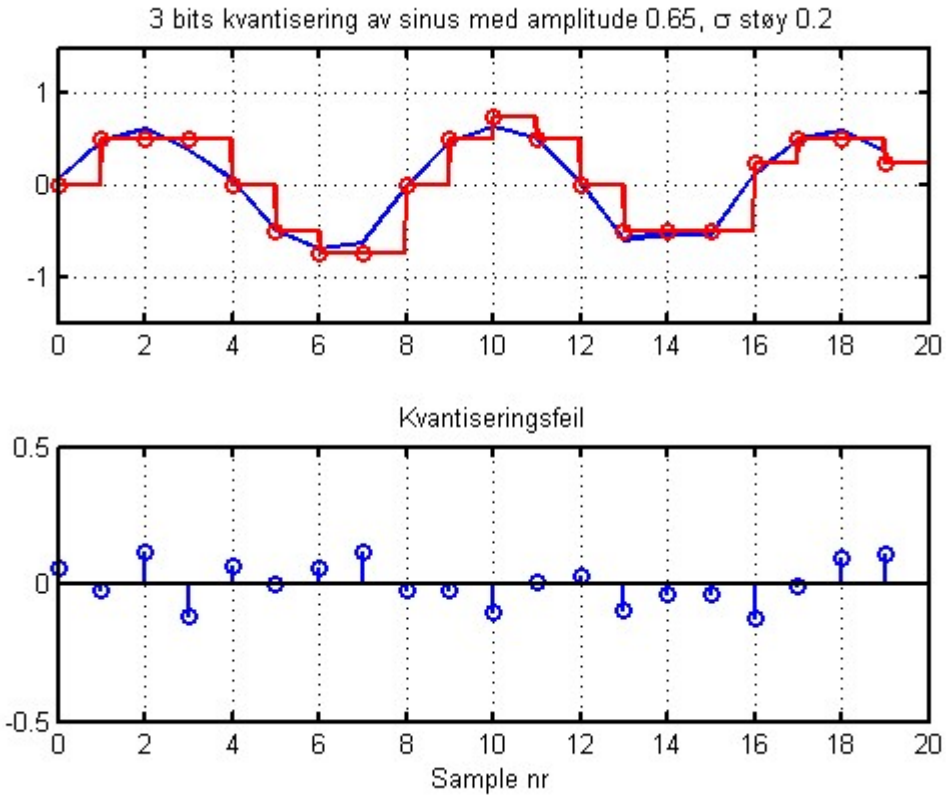
- x(t) = Acos(·)
- Effekt, P_S = effekt i signal = $A^2/2$
- Spiss-til-spissverdi: D = 2A

$$SNR = 10\log(A^2/2) + 6B - 20\log(2A) + 10.8$$

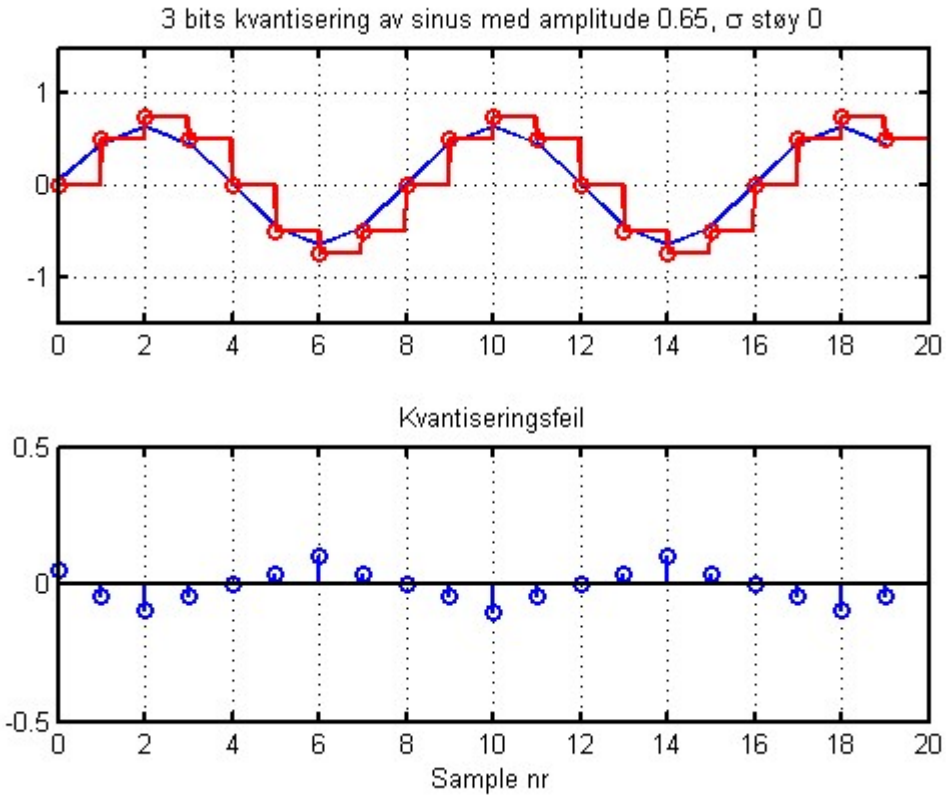
$$SNR = 20\log(A) - 3 + 6B - 20\log(A) - 6 + 10.8 = 6B + 1.8$$

- Eksakt resultat: SNR = 6B+1.76 dB

3-bits kvantisering av sinus i støy: ~ støyaktig kvantiseringsfeil



Kvantisering av sinus uten støy: periodisk kvantiseringsfeil



Regulær kvantiseringsstøy «Limit cycles»

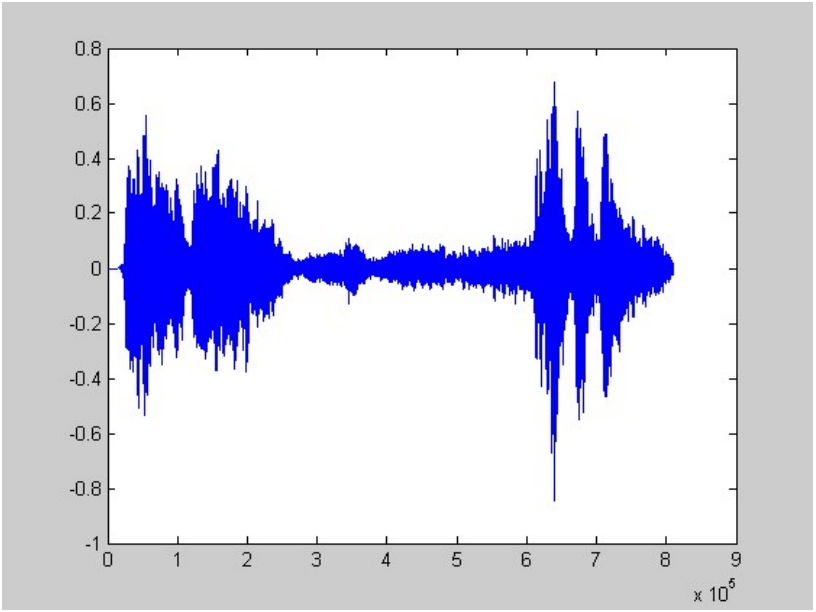
- DAC og regulær sinus: periodiske støykomponenter.
  - Direkte digital syntese (DDS) gir alltid dette

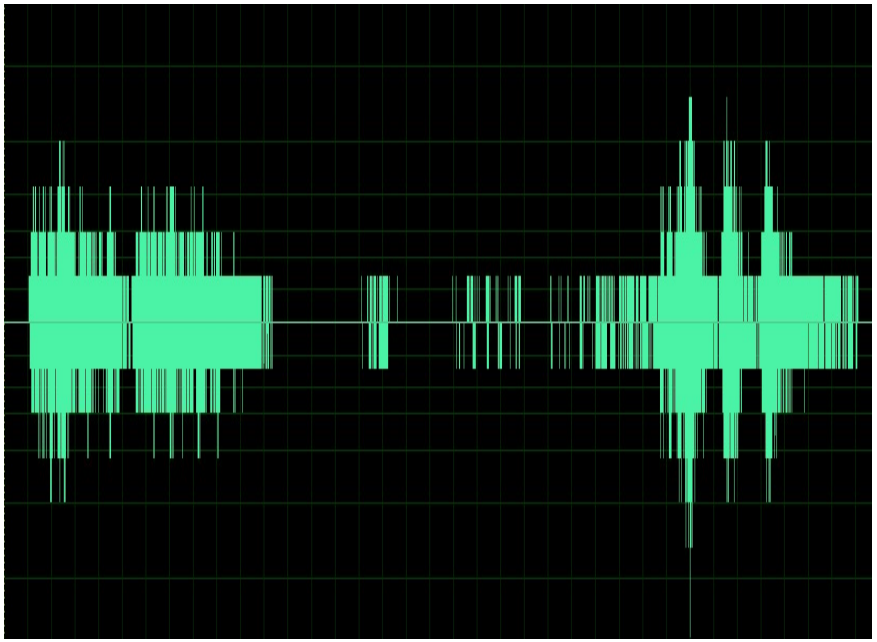
- ADC og periodiske signaler
  - ikke ukorrellert kvantiseringsstøy
- ADC og lave signaler:
  - ADC låser seg på et nivå; trenger litt støy

Beethovens 5. symfoni

16 bit kvantisering
2^{16}= 65536 nivåer

4 bits kvantisering
2^4=16 nivåer





Tid

ADC og dithering

- Analogt signal med liten amplitude
  - Nesten ikke avvik fra konstant signal etter kvantisering
- Legger på litt støy – dither (=skjelve, dirre)
  - Følger opprinnelig signal bedre

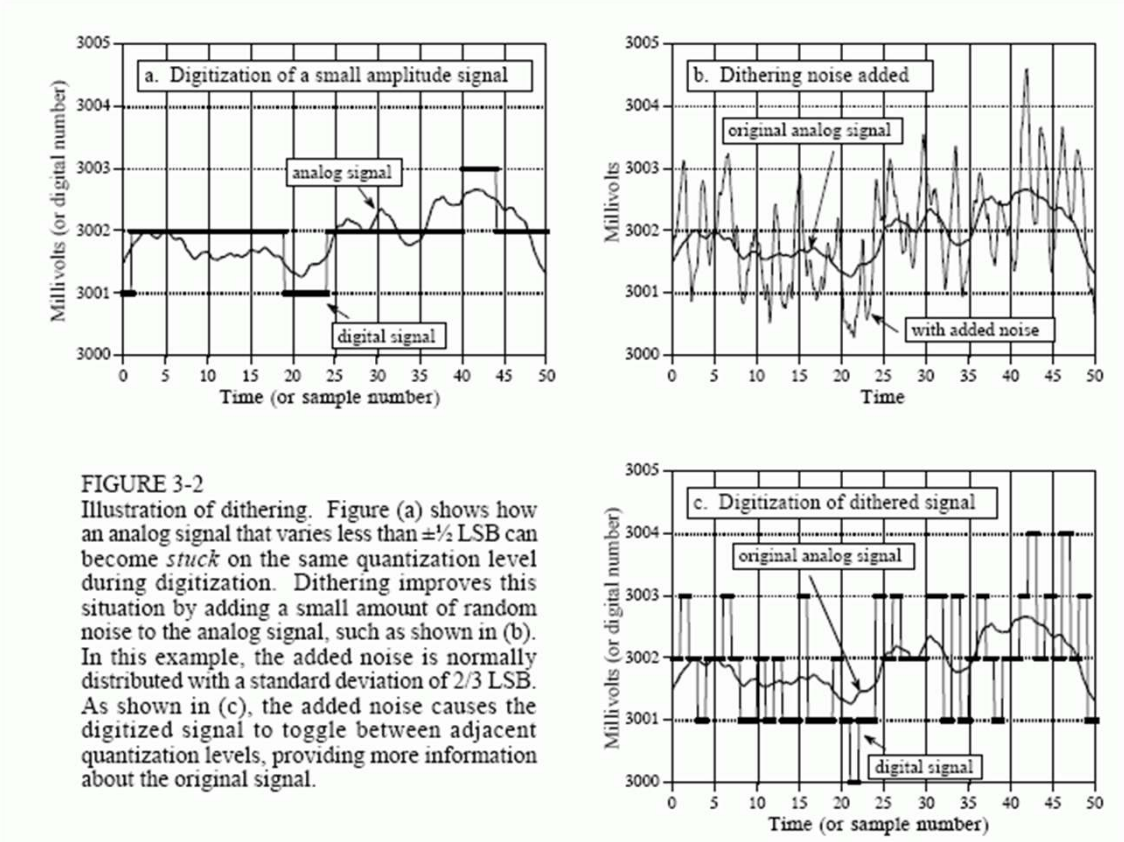


FIGURE 3-2
Illustration of dithering. Figure (a) shows how an analog signal that varies less than ±½ LSB can become *stuck* on the same quantization level during digitization. Dithering improves this situation by adding a small amount of random noise to the analog signal, such as shown in (b). In this example, the added noise is normally distributed with a standard deviation of 2/3 LSB. As shown in (c), the added noise causes the digitized signal to toggle between adjacent quantization levels, providing more information about the original signal.

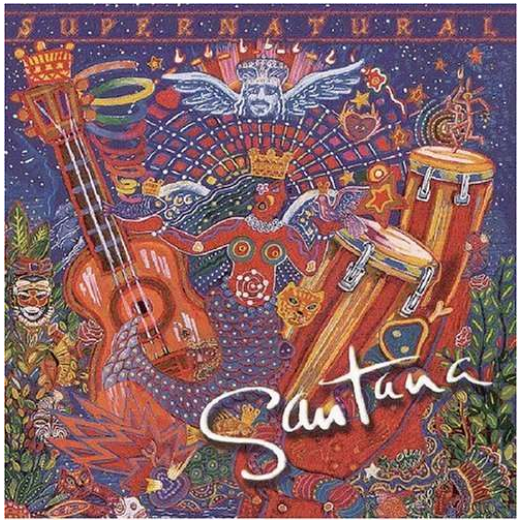
Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

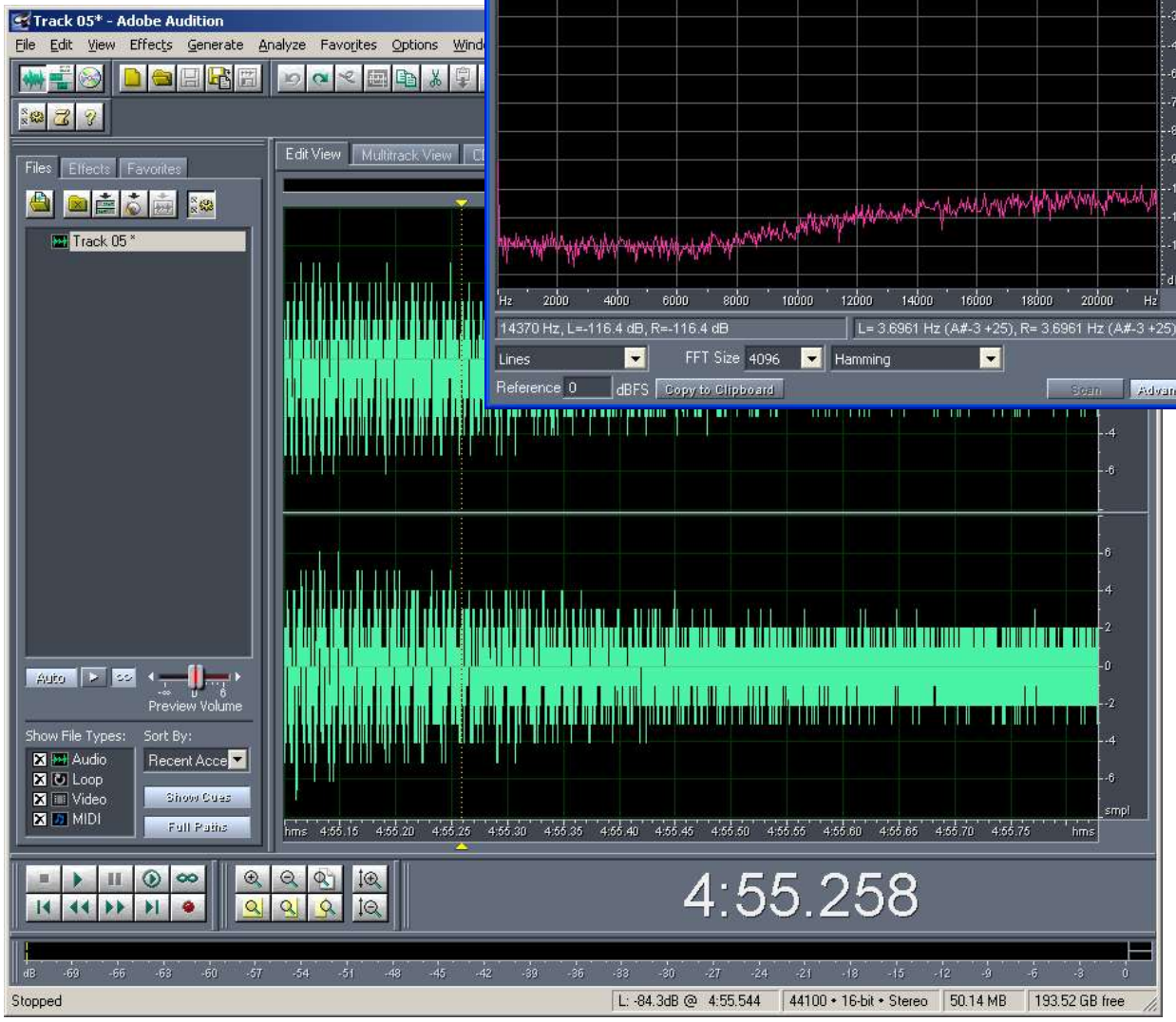
Dårlige CD-plater: Dither

- Tidlige CD-plater: avrundingsproblem.
- Også under prosessering og miksing med heltalls-aritmetikk
- Ikke som avrunding i kassa på Rema
- => hørbar forringelse i de svake partiene.
  - Eks: pianotoner som dør ut
- Paradoks: må legge til kontrollert støy, dither, ved produksjon av CD-en.
  - Støyforming så dens energi er mest i diskanten

Dither

- Santana: Smooth, 1999
- Slutten av låten
- Maks +/- 3
- Trekant-fordeling
- Støyforming





Analog til digital-omforming

- Kvantisering
  - Kvantiseringsstøy: SNR [db] $\propto$ 6 B [bits]
  - Overstyring
  - Dithering: Ved periodisk signal og ved lavt nivå
- Samplingsteoremet
  - Utledning ved analog/digital Fourier
  - Aliasing
  - Samplingsskop og båndpassampling

Samplingsteoremet

- Relativ frekvens
  - 0
  - 0.09
  - 0.31
  - 0.95

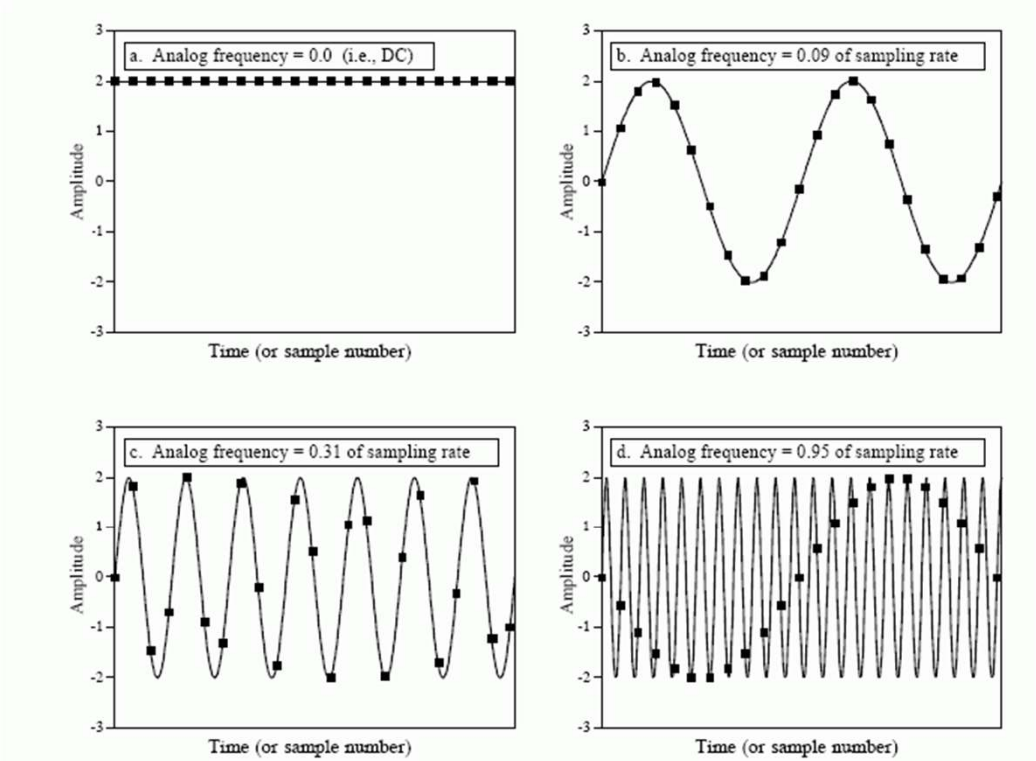
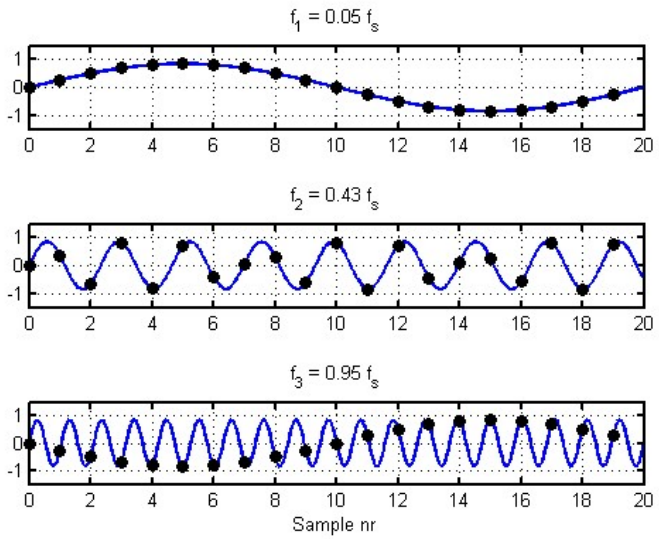


FIGURE 3-3
Illustration of proper and improper sampling. A continuous signal is sampled *properly* if the samples contain all the information needed to recreate the original waveform. Figures (a), (b), and (c) illustrate *proper sampling* of three sinusoidal waves. This is certainly not obvious, since the samples in (c) do not even appear to capture the shape of the waveform. Nevertheless, each of these continuous signals forms a unique one-to-one pair with its pattern of samples. This guarantees that reconstruction can take place. In (d), the frequency of the analog sine wave is greater than the Nyquist frequency (one-half of the sampling rate). This results in *aliasing*, where the frequency of the sampled data is different from the frequency of the continuous signal. Since aliasing has corrupted the information, the original signal cannot be reconstructed from the samples.

http://www.dspguide.com/ch3/2.htm

Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

Aliasing: første og siste eksempel lar seg ikke skille fra hverandre



- Relativ frekvens
  - 0.05
  - 0.43
  - 1-0.05=0.95
- Bortsett fra fortegn gir to av dem samme samples

Aliasing

Samplingsfrekvens: S = 1

Nyquist-frekvens: S/2

- Frekvenser < S/2: OK
- f=0.95 S alias -> 0.05 S
- Alle frekvenser aliaser til intervallet 0 …S/2

Nedre diagram er fase (mindre viktig)

http://www.dspguide.com/ch3/2.htm

Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

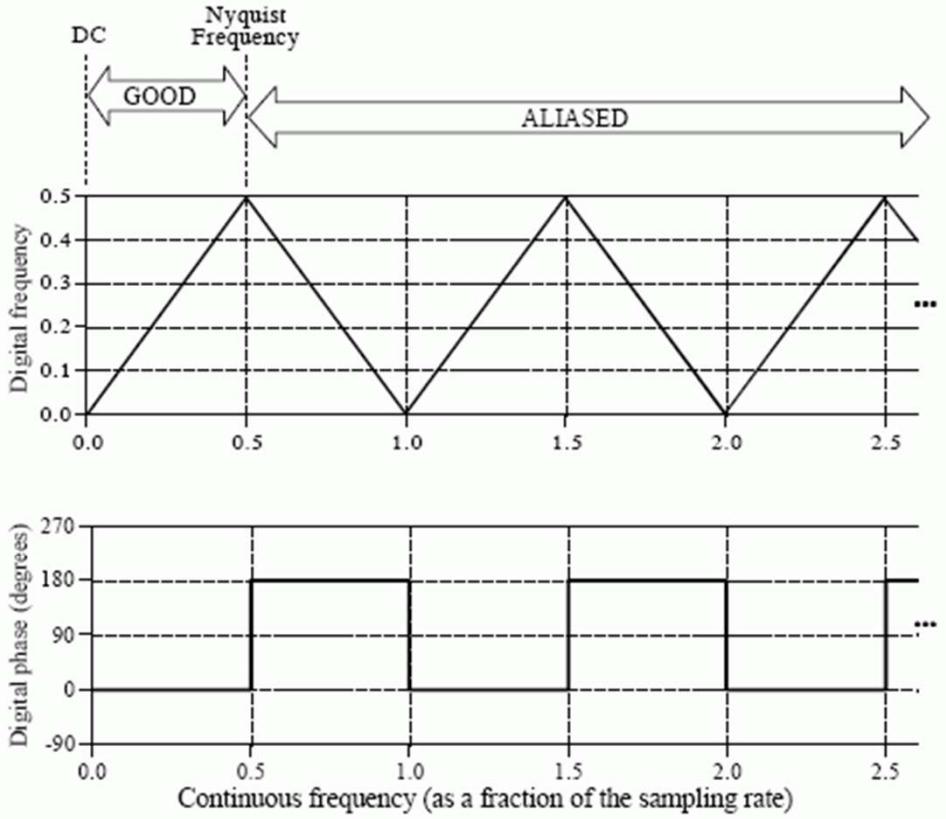


FIGURE 3-4
Conversion of analog frequency into digital frequency during sampling. Continuous signals with a frequency less than one-half of the sampling rate are directly converted into the corresponding digital frequency. Above one-half of the sampling rate, aliasing takes place, resulting in the frequency being misrepresented in the digital data. Aliasing always changes a higher frequency into a lower frequency between 0 and 0.5. In addition, aliasing may also change the phase of the signal by 180 degrees.

Over- og under-sampling

a) og b): analog tid og frekvens

c) og d): S=3 fmax

e) og f) S=1.5 fmax

d) Ingen overlapp mellom aliaser

Rekonstruksjon er mulig ved lavpassfiltrering

e) Overlapp

Rekonstruksjon **er ikke** mulig

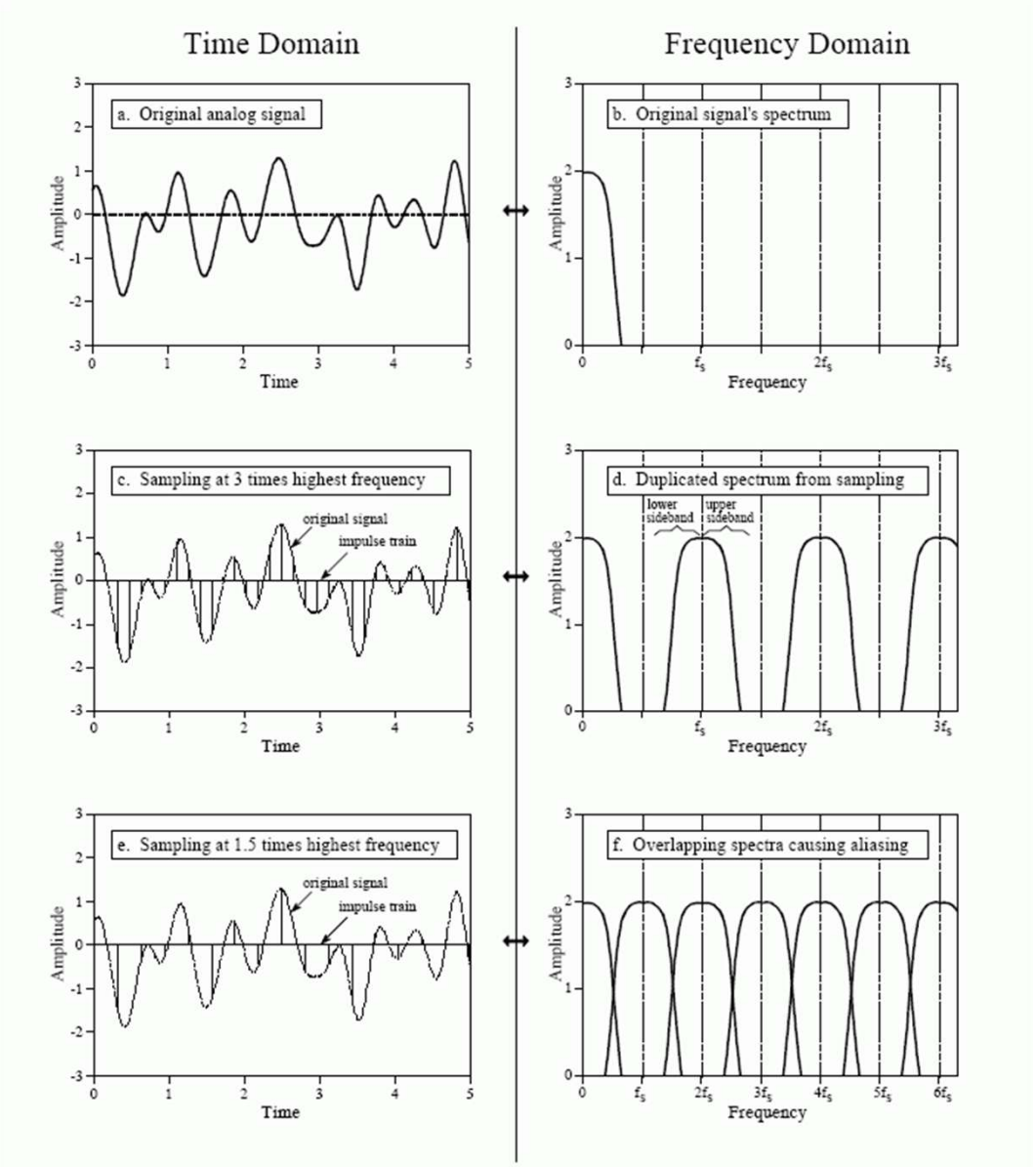


FIGURE 3-5
The sampling theorem in the time and frequency domains. Figures (a) and (b) show an analog signal composed of frequency components between zero and 0.33 of the sampling frequency, f_s. In (c), the analog signal is sampled by converting it to an impulse train. In the frequency domain, (d), this results in the spectrum being duplicated into an infinite number of upper and lower sidebands. Since the original frequencies in (b) exist undistorted in (d), proper sampling has taken place. In comparison, the analog signal in (e) is sampled at 0.66 of the sampling frequency, a value exceeding the Nyquist rate. This results in aliasing, indicated by the sidebands in (f) overlapping.

Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

http://www.dspguide.com/ch3/3.htm

Sinuser og periodiske signaler

- Sinus med frekvens f_0:
  - Høyeste (= eneste) frekvens $B=f_0$ ⇔ sampling: $S \geq 2f_0$
- Minst to samples pr periode
  - NB! Farlig med akkurat to pr periode: kan treffe nullgjennomganger: unngå kritisk sampling
- Oversamplet, $S>2f_0$: Impulser ved $\pm\ f_0$
- Undersamplet, $S<2f_0$: Aliasing
  Impulser i $\pm\ f_a$, $|f_a| < S/2$
  - Finner f_a ved å trekke et helt antall S fra f_0: $f_a=f_0-N\cdot S$ der N er slik at $|f_a| < S/2$: Alltid: $f_a \leq f_0$
    - Denne formelen er litt uvanlig, må itereres. Som regel er N liten så det kan gjøres enkelt.

Aliasing - eksempler

Sinus på f_0=100 Hz

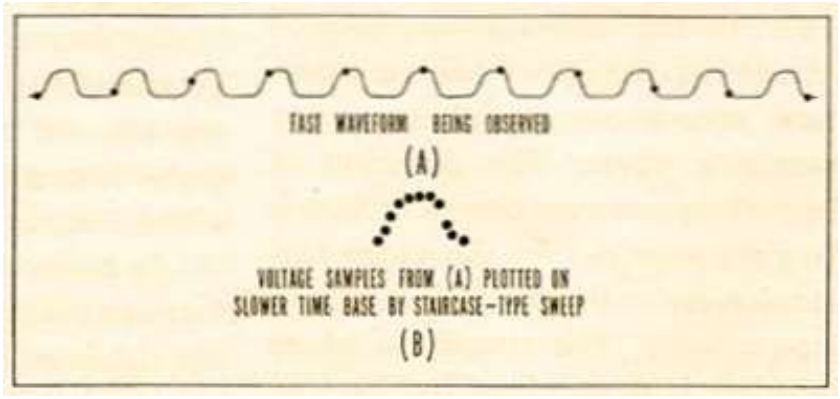
1. S=300 Hz:
   - Ingen aliasing
2. S=80 Hz < $2f_0$:
   - $f_a=f_0-N\cdot S=100-N\cdot 80=20$ (N=1)
3. S=60 Hz < $2f_0$:
   - $f_a=f_0-N\cdot S=100-N\cdot 60=-20$ (N=2) [N=1 $\Rightarrow$ 40 >S/2]
   - Nå blir opprinnelige positive frekvenser aliaset til negative og omvendt, dvs fase reversering.

Samplingsskop: utnytter aliasing

- Anta bare ett periodisk signal – gjentar seg: kan måle mange ganger
- Båndbegrenset
- Får en strukket versjon av signalet ved å sample litt senere for hver periode
  - Samme prinsipp:
    3D avbildning av hjertet med ultralyd gatet mot EKG: endel av bildet i hver periode
  - Stroboskop

Samplingsskop

- Undersampling av periodisk bølgeform
  - Høy frekvens samples langsomt ved å bruke mer tid



  - The Signal Sampling Principles, March 1962, Hewlett Packard Journal
  - http://www.hpmemory.org/wb_pages/wall_b_page_02.htm

Samplingsskop

- Signal: $1+\cos(2\pi f_0 t)$: f_0=100 MHz (fig a)
- Nyquistsampling S>200 MHz
  - Lav båndbredde så man kan undersample i stedet
- Eks: S=98 MHz, aliaser til 100-98=2 MHz (fig b)
- Tidsstrekking med $\alpha=f_0/f_a = f_0/(f_0-S)$ = 100/2=50
  - Jo mer S $\rightarrow$ f_0, jo lenger tidsstrekking

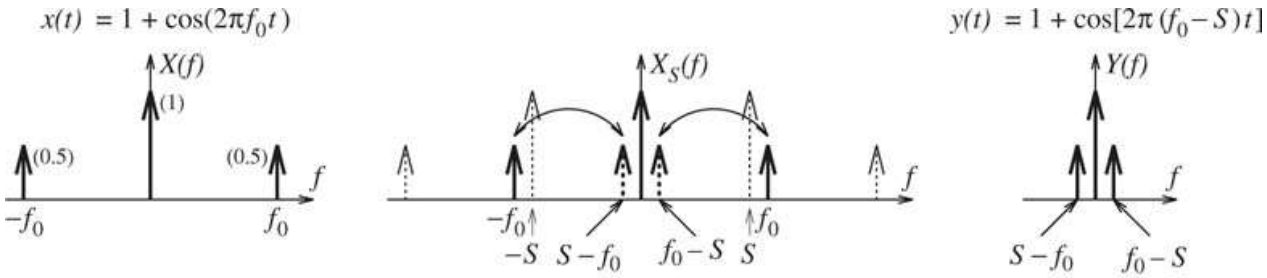


FIGURE 7.5 The principle of the sampling oscilloscope. A high-frequency sinusoid is sampled at a rate far less than the Nyquist rate. The spectrum of the sampled signal corresponds to a much lower frequency and the signal can be displayed on a conventional oscilloscope