

UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

FYS3220 – Lineær kretselektronikk

Digital til analog

Sverre Holm

Digital til analog-omforming

- Interpolasjon ved gjenvinning
  - 0’te-ordens hold
  - Sinc-interpolasjon
  - 1ste ordens hold
- Filtrering ved ADC og DAC
- Oversampling og delta-sigma ADC

Gjenvinning: 0-te ordens hold

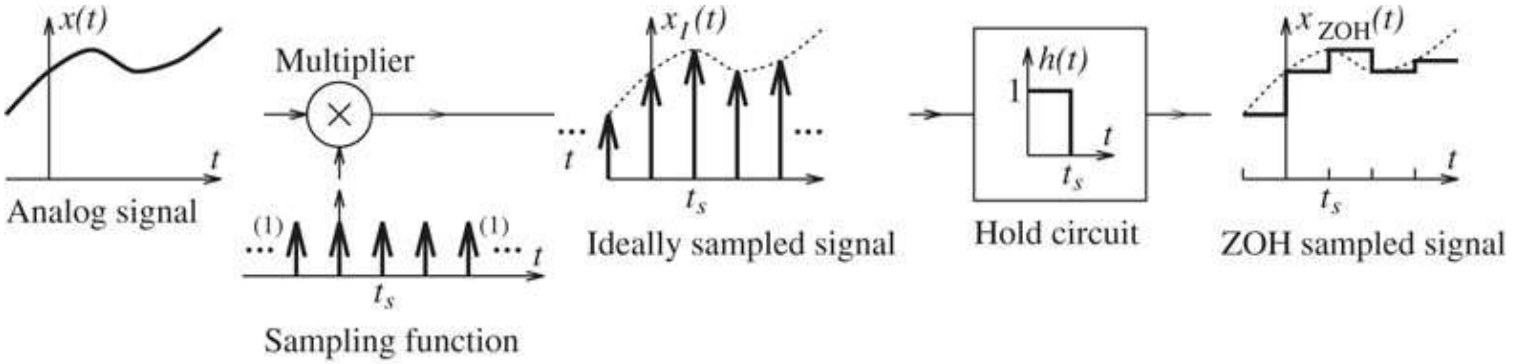


FIGURE 7.8 Zero-order-hold sampling is equivalent to ideal sampling followed by a hold operation. Multiplying an analog signal by a periodic-impulse train (the ideal sampling function) results in a train of impulses whose strengths match the values of the analog signal at the sampling instants. The hold operation (convolution of the impulses with a rectangular pulse) produces a staircase approximation of the analog signal

- Ekvivalent med sampling fulgt av filter
- Viktigere ved gjenvinning med D/A enn ved sampling med A/D

Ambardar, Digital Signal Processing, A Modern Introduction

Hold-filter

a), b): Samples, repeterte spektra

c) Etter 0-te ordens hold

d) Frekvensrespons:

EQUATION 3-1
High frequency amplitude reduction due to the zeroth-order hold. This curve is plotted in Fig. 3-6d. The sampling frequency is represented by f_s. For $f = 0$, $H(f) = 1$.

$$H(f) = \left| \frac{\sin(\pi f / f_s)}{\pi f / f_s} \right|$$

e) Invers frekvensrespons for å oppheve 0-te ordens hold

f), g): perfekt rekonstruert

Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

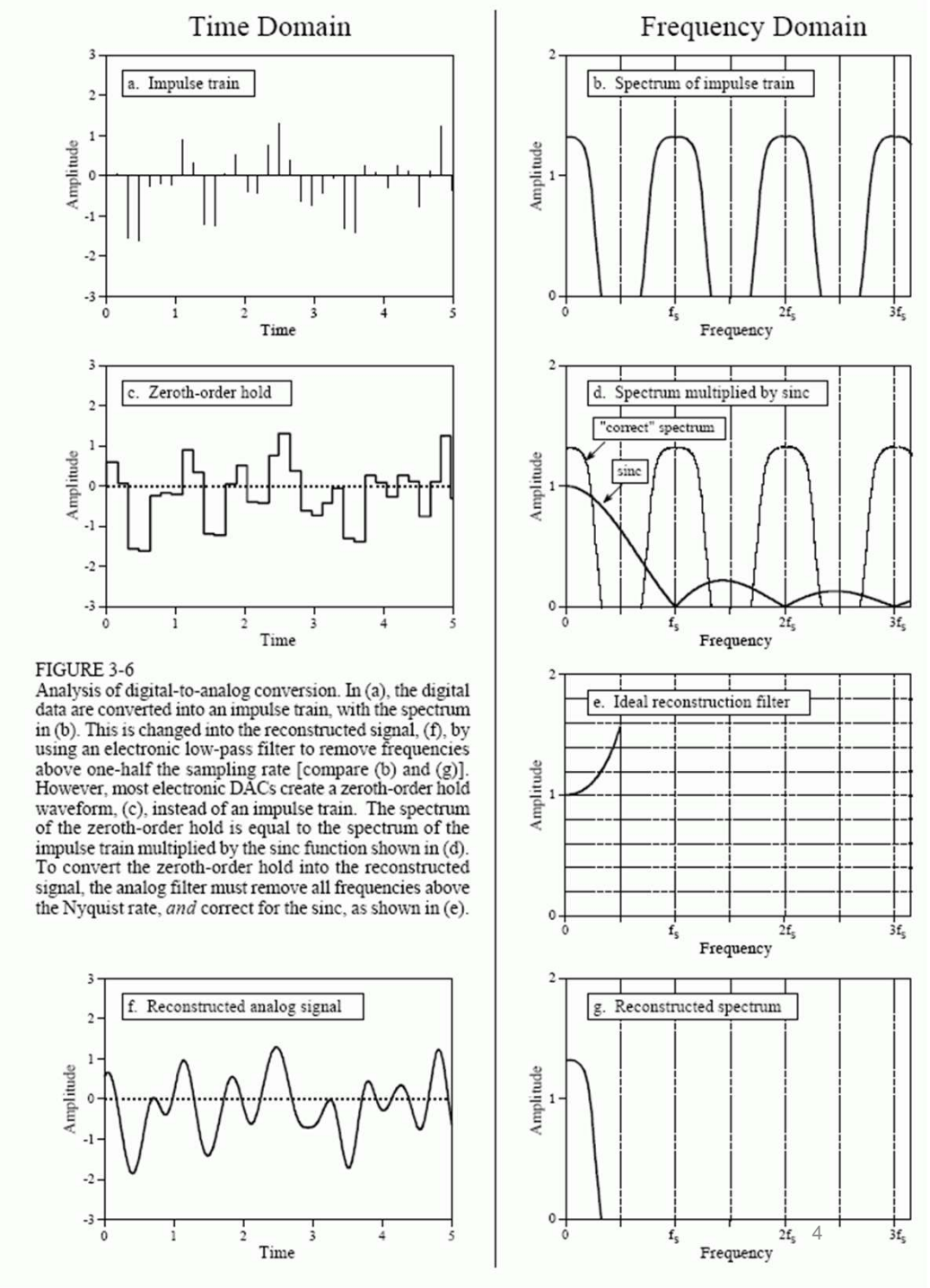


FIGURE 3-6
Analysis of digital-to-analog conversion. In (a), the digital data are converted into an impulse train, with the spectrum in (b). This is changed into the reconstructed signal, (f), by using an electronic low-pass filter to remove frequencies above one-half the sampling rate [compare (b) and (g)]. However, most electronic DACs create a zeroth-order hold waveform, (c), instead of an impulse train. The spectrum of the zeroth-order hold is equal to the spectrum of the impulse train multiplied by the sinc function shown in (d). To convert the zeroth-order hold into the reconstructed signal, the analog filter must remove all frequencies above the Nyquist rate, *and* correct for the sinc, as shown in (e).

0-te ordens hold

- Respons til hold-filteret,
  - h(t) = rect[t-0.5t_s/t_s], dvs 1 for 0 < t < t_s
- Dette filteret er analogt og må analyseres med kontinuerlig Fourier transform
- Hold-filterets frekvensrespons:

$$H(f) = t_s \mathrm{sinc}(f t_s) e^{-j\pi f t_s} = \frac{1}{S}\mathrm{sinc}(f/S) e^{-j\pi f t_s}, \mathrm{sinc(x)} = \sin(\pi x)/(\pi x)$$

0-te ordens hold

- https://en.wikipedia.org/wiki/Zero-order_hold

0-te ordens hold ⇔ filter

$$H(f) = t_s \mathrm{sinc}(f t_s) e^{-j\pi f t_s} = \frac{1}{S}\mathrm{sinc}(f/S) e^{-j\pi f t_s}, \mathrm{sinc(x)} = \sin(\pi x)/(\pi x)$$

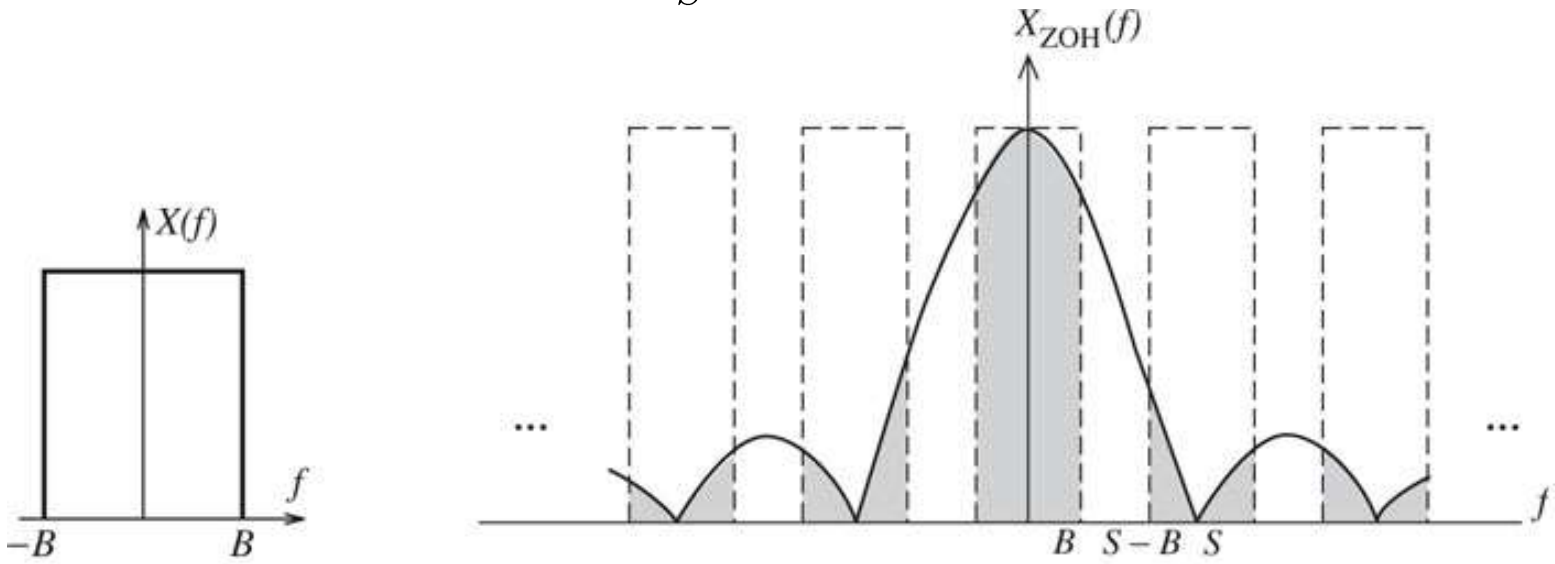


FIGURE 7.9 Illustrating the spectrum of a zero-order-hold sampled signal. The spectrum of the analog signal is $X(f)$. The spectrum of the ideally sampled signal (images of $X(f)$) is multiplied by the spectrum of the hold system (a sinc with a main lobe and sidelobes) to produce the spectrum of the ZOH signal (shown shaded). Note how the sinc shape results in distortion of the images

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0-te ordens hold: sinc-forvrengning

- Må motforvrenge for å oppheve sinc-funksjonen
  - Ideelt LP for å gjenvinne analogt signal fra samplene
    - Fulgt av kompensasjonsfilter
  - Eks: D/A-konverter i digital musikk-spiller

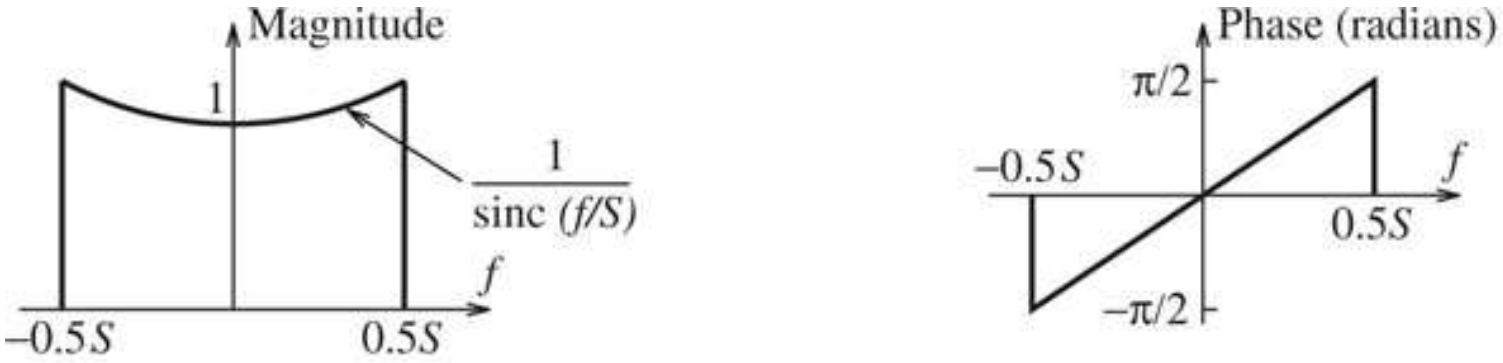


FIGURE 7.10 Spectrum of a filter that compensates for sinc distortion. To undo its effects, the filter gain is chosen as the reciprocal of the sinc (in the principal range).The filter is chosen to have linear phase

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0-te ordens hold: motforvrengning (eks)

- Hvor mange dB er maks kompensasjon i et system som er samplet ved Nyquist-raten (worst-case)?

$$H(f) = t_s \mathrm{sinc}(f t_s) e^{-j\pi f t_s} = \frac{1}{S}\mathrm{sinc}(f/S) e^{-j\pi f t_s}, \mathrm{sinc(x)} = \sin(\pi x)/(\pi x)$$

- sinc(0)=1
- sinc(f_{max}/S) = sinc(0.5) = sin(π/2)/(π/2) = 2/π
- Fall i dB: 20log[(2/π)/1] ≈ -3.9 dB

- Kompensasjon er maks 3.9 dB

Sampling – gjenvinning

- Hvordan skal man fylle inn mellom samplene – interpolasjon?

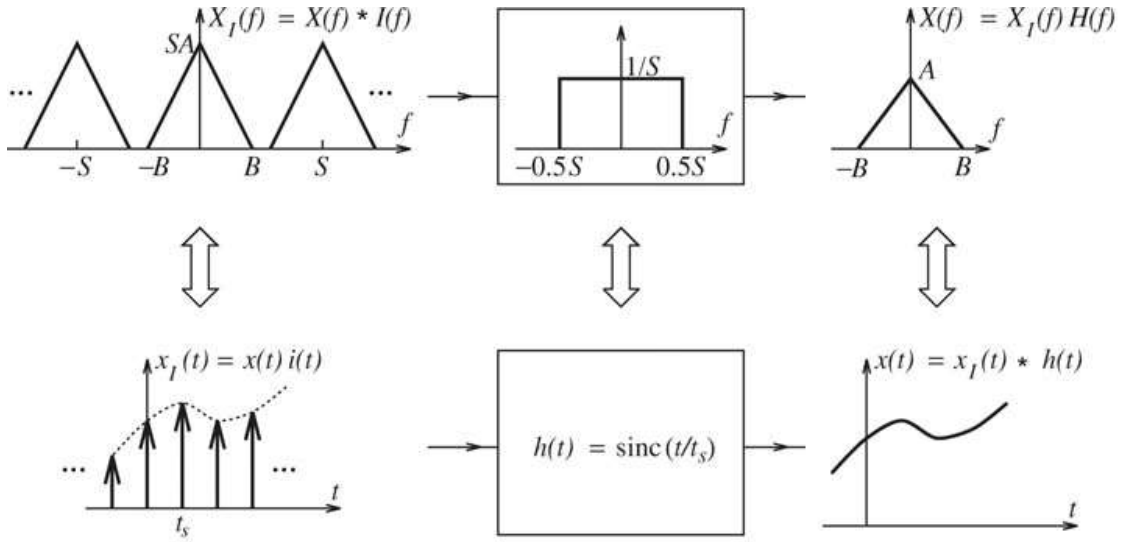


FIGURE 7.11 Recovery of an analog signal from its sampled version. Ideal recovery uses an ideal lowpass filter that extracts the central period of the periodic spectrum of the sampled signal. The equivalent operation in the time domain corresponds to the convolution of the sampled signal with the impulse response of the ideal filter. If the sampling rate exceeds the Nyquist rate, the central period contains just the spectrum of the original signal, and we recover the original signal

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Interpolasjonsfilter

- Ideelt lavpassfilter - ”Mursteinsfilter”
  - Analogt filter: H(f)=1 for |f|<S/2 ⇔ h(t)=sinc(t/t_s)

- Analog konvolusjon: x(t)=x_I(t) * h(t):

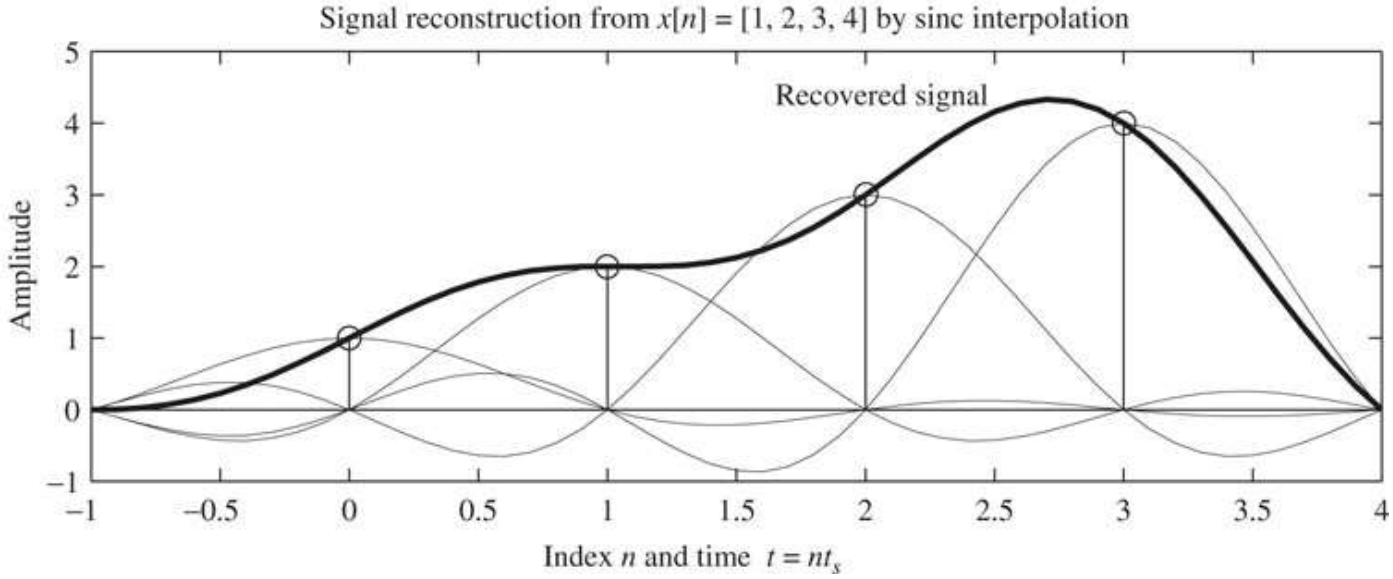
$$x(t) = \int_\tau x_I(\tau)h(t-\tau)d\tau = \int_\tau \sum_n x[n]\delta(\tau - nt_s)h(t-\tau)d\tau$$

- Forflyttede versjoner av sinc-funksjoner over all tid

Sinc interpolasjon

$$x(t) = \sum_n x[n]h(t - nt_s) = \sum_n x[n]\mathrm{sinc}\left(\frac{t-nt_s}{t_s}\right)$$

FIGURE 7.12 Ideal recovery of an analog signal by sinc interpolation. The recovered signal is the sum of sinc functions centered at the sampling instants. The zeros of these sinc functions occur at multiples of the sampling interval



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Merk:

- I et sampletidspunkt, $t=nt_s$: bare bidrag fra en sample
- Må kjenne alle samples, $-\infty < n < \infty$, for gjenvinning
- Sinc er korrekt, alle andre interpolatorer måles mot den

Uendelig lang sinc i praksis

- Digital audio (CD), fs = 44100 Hz = 1/ts, 16 bit
- Endelig lengde: tilnærming:
  - Hvis sinc-ens verdi blir < 1 bit holder det i hvertfall
  - 16 bit ⇔ +/- 32768
- $|\mathrm{sinc}(t/ts)| = |\sin(\pi t/ts)/(\pi t/ts)| < ts/(\pi t) = 1/32768$
- Det gir t = 0.237 s for ts = 1/44100
- Ingen vits med filter som er lenger enn +/- 0.237 sek
- Dette er er meget langt filter:
  - Orden N = 2*0.237*44100 = 20903
  - Mye lenger enn i praksis!

Tilnærminger til sinc-interpolasjon

Krav:

- Må treffe eksakt i sampletidspunktene, $t=nt_s$
  - $h_i(t)=1$, $t=0$ og $h_i(t)=0$ for $t=nt_s$
- Må unngå at den er ∞ lang
- $h_i(t)$ må være absolutt integrerbar så den ikke blir ∞ mellom sampletidspunkter

0. eller 1. ordens hold

- Trinn-interpolasjon (0-te ordens hold), allerede analysert:

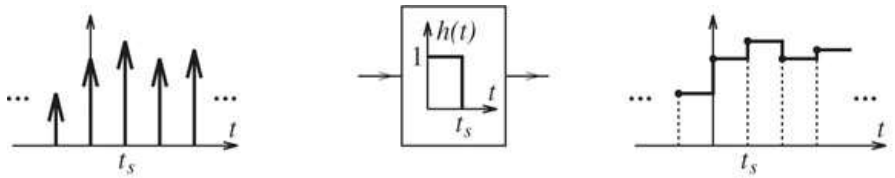


FIGURE 7.13 Step interpolation is just a zero-order-hold operation on an ideally sampled signal. It produces a staircase approximation of the original analog signal

- Lineær interpolasjon: trenger en sample inn i fremtiden

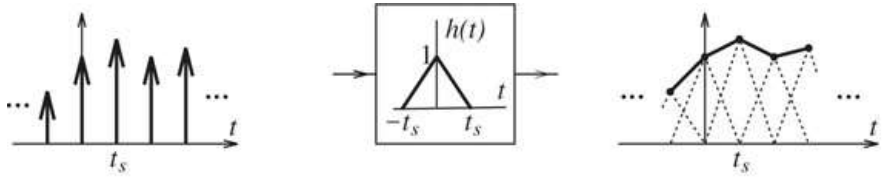
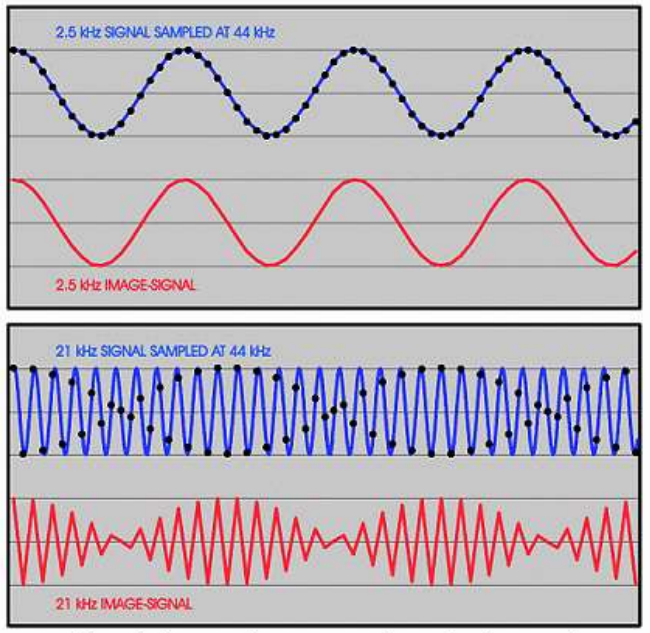


FIGURE 7.14 Linear interpolation is equivalent to using tri functions centered at the sampling instants of an ideally sampled signal to produce a piecewise linear approximation of the original analog signal

- Generelt: $w(t)\mathrm{sinc}(t/t_s)$ der $w(t)$ er en vindusfunkjson
  - $w(0)=1$ og $w(t) \rightarrow 0$ for store $|t|$

The Analoguer - A Remedy for Digitalitis

- Hvilket problem forsøkes løst her og er det egentlig et problem?
- Sitat:
- *"The major anomaly of the CD is introduced by the relatively low sampling frequency of 44 kHz. Although this sampling frequency allows us to record signals up to 22 kHz the upper frequencies are not very well presented.*
- *Figure 1 compares the sampling of a 2.5 kHz and of a 21 kHz waveform. After sampling of the original signals I connected the consecutive measurement-points by straight lines.* ?
- *The image-signal of the 2.5 kHz signals thus constructed equals the original signal pretty well.*
- *The image of the 21 kHz signal, however, continuously varies in amplitude; it "wobbles". This phenomenon can be seen with all the higher frequencies above 15 kHz and is an inherent property of the system.*
- *... the continuously varying amplitudes in combination with non-linearities of our reproduction system and our ears introduce lower frequency by-products that might well be discernible."*
  - http://www.headwize.com/projects/showfile.php?file=meier4_prj.htm



Effect of 44kHz Sampling Rate on 2.5kHz and 21kHz Signals

Ikke en analyse av sampling, men av lineær interpolator!

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- Filtrering ved ADC og DAC
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Analoge Filtre ved ADC og DAC

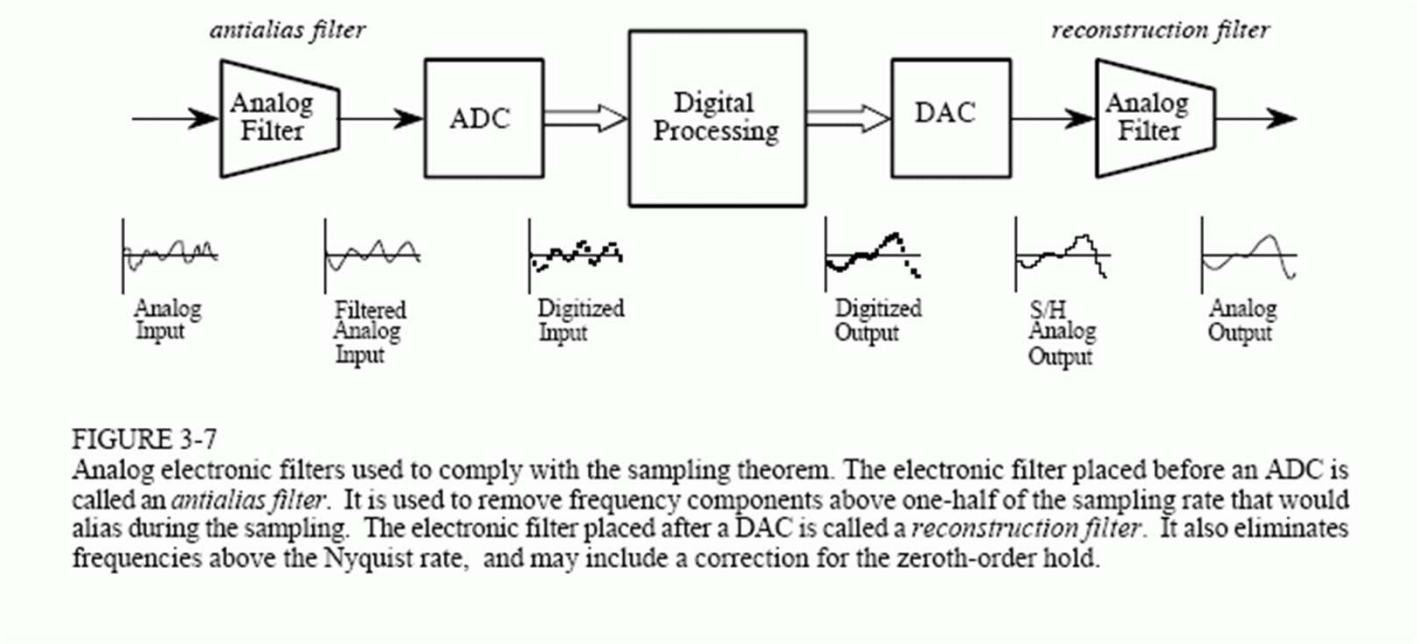


FIGURE 3-7
Analog electronic filters used to comply with the sampling theorem. The electronic filter placed before an ADC is called an *antialias filter*. It is used to remove frequency components above one-half of the sampling rate that would alias during the sampling. The electronic filter placed after a DAC is called a *reconstruction filter*. It also eliminates frequencies above the Nyquist rate, and may include a correction for the zeroth-order hold.

http://www.dspguide.com/ch3/4.htm

Fig: The Scientist and Engineer's Guide to
Digital Signal Processing, Steven W. Smith

Anti-aliasing filter

- Analogt filter
- Båndbegrensning av input signal: bare f < S/2
  - I praksis kan man ikke lage ideelle mursteinsfiltre
  - Blir derfor noe aliasing
- Mulig kriterium: energi over S/2 skal være mindre enn kvantiseringsstøyen
  - Kan avlede filterkrav av dette
- Kan og sette S litt høyere enn ideell verdi

Standard-filtre

- Hvor skarpt faller det av i transisjonbåndet?
- Passbånd?
  - Log: passbånd ser bra ut
  - Lin: ikke fullt så bra likevel!
- Stoppbånd?
  - Krav: Fs=10 kHz & dempe .01 @ 5 kHz
  - Tsjebysjeff, 8 pol gir smalest transisjonbånd: f=1.35
  - La derfor 1.35 svare til 5 kHz
  - Fcut = 5kHz/1.35 = 3.7 kHz er i praksis høyeste brukbare frekvens:
  - 3.7-5 kHz: 1.3 kHz bortkastet
  - 5-6.3 kHz vil aliase inn
  - 6.3/3.7 = 1.7 blir i praksis fcut
  - Demper der med 1300
- I praksis: f >0.4 fs er bortkastet

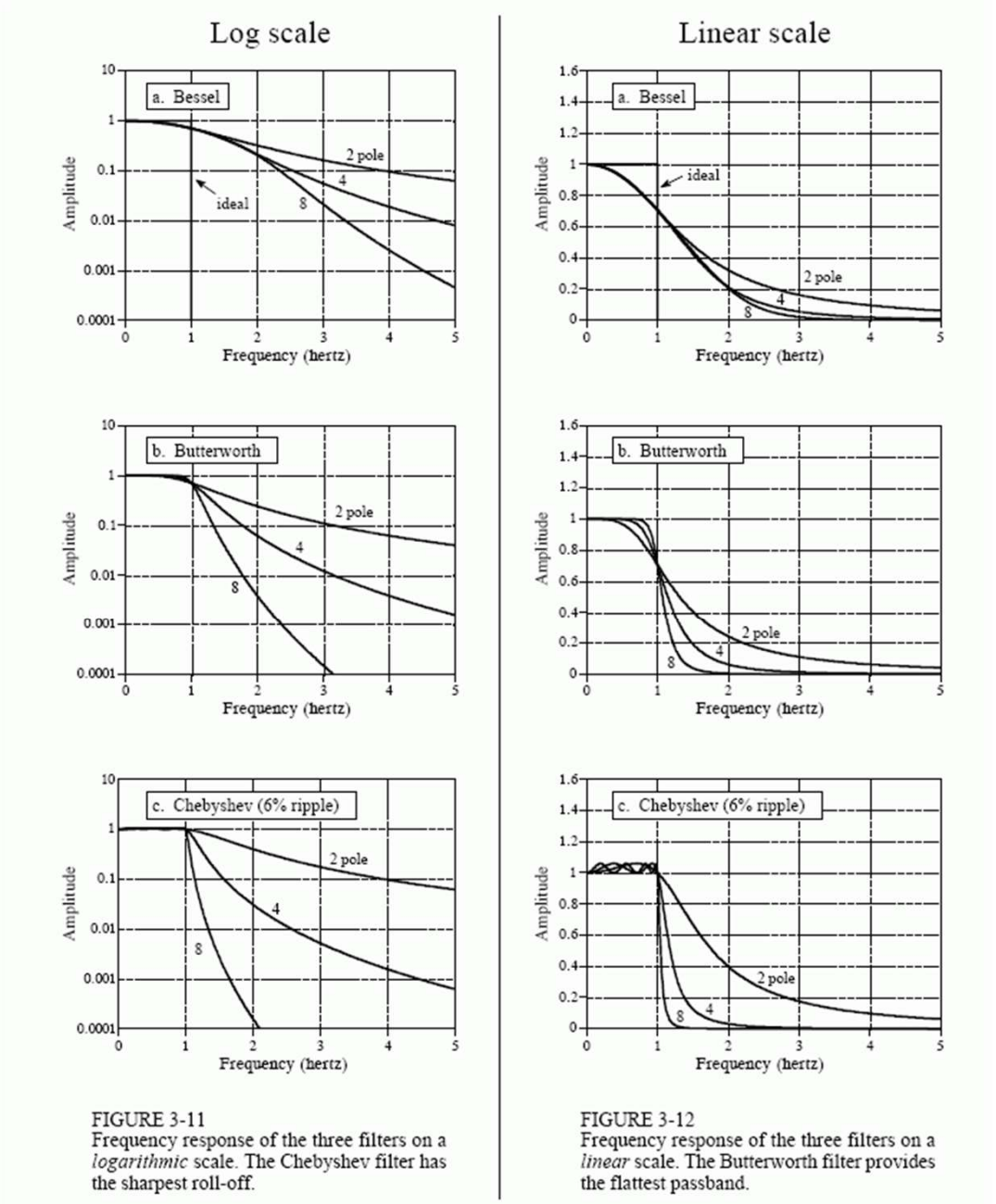


FIGURE 3-11
Frequency response of the three filters on a *logarithmic* scale. The Chebyshev filter has the sharpest roll-off.

FIGURE 3-12
Frequency response of the three filters on a *linear* scale. The Butterworth filter provides the flattest passband.

Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

Valg av antialias-filter

Hvis pulssignal skal samples og HF fjernes

- Bessel bevarer pulsform mye bedre enn Tsjebysjeff
- Kan være viktig ved medisinske signaler

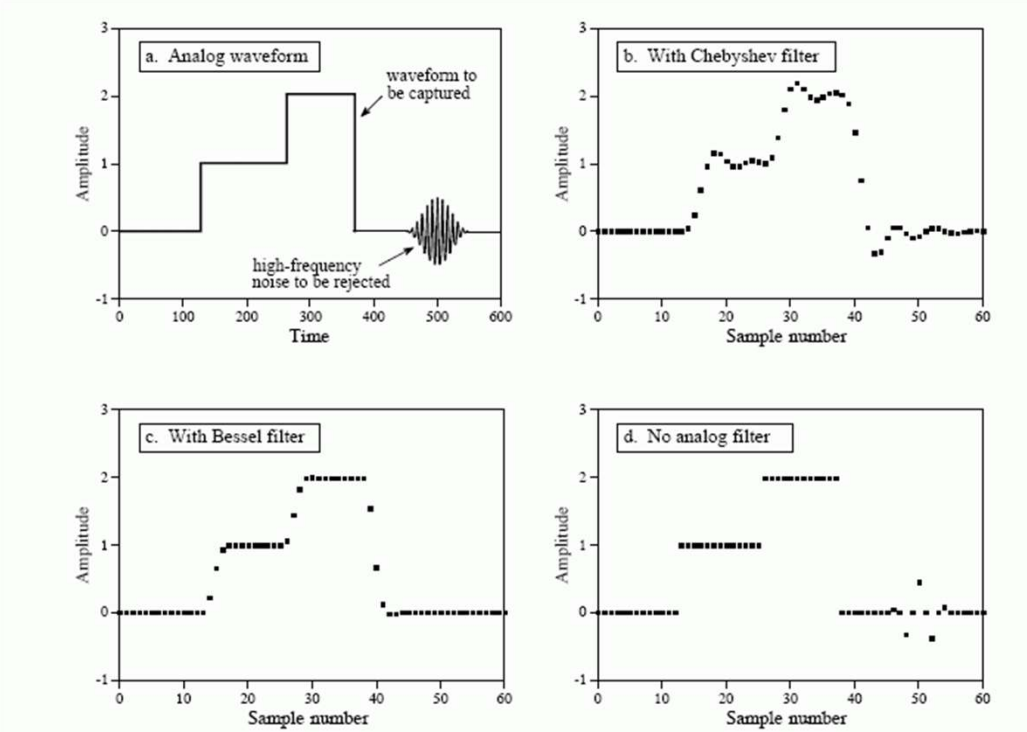


FIGURE 3-15
Three antialias filter options for time domain encoded signals. The goal is to eliminate high frequencies (that will alias during sampling), while simultaneously retaining edge sharpness (that carries information). Figure (a) shows an example analog signal containing both sharp edges and a high frequency noise burst. Figure (b) shows the digitized signal using a *Chebyshev filter*. While the high frequencies have been effectively removed, the edges have been grossly distorted. This is usually a terrible solution. The *Bessel filter*, shown in (c), provides a gentle edge smoothing while removing the high frequencies. Figure (d) shows the digitized signal using *no antialias filter*. In this case, the edges have retained perfect sharpness; however, the high frequency burst has aliased into several meaningless samples.

Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

Rekonstruksjonsfilter

- Analogt filter
- Ta ut sentrale delen av spektret, |f|<S/2
- Evt. korrigere for 0-te ordens hold i D/A
- Oversampling (eks 4x)
  - Kan virke bortkastet
  - Gjør kravet til analogt filter mye lettere (Eks, neste slide)
  - Lenger avstand mellom repeterte spektra ⇔ glattere trinn

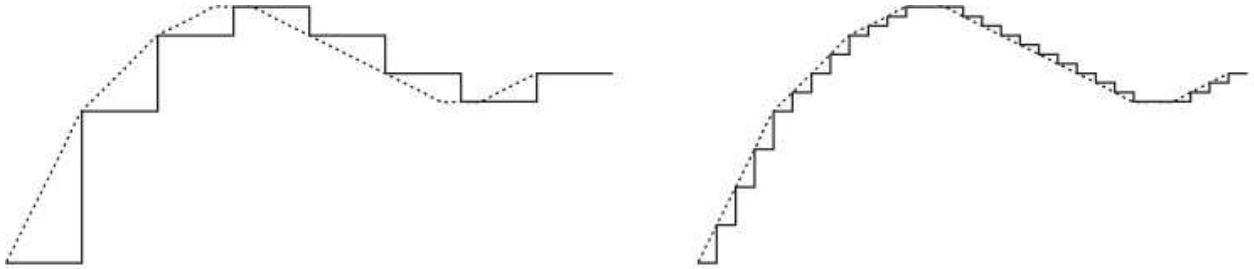
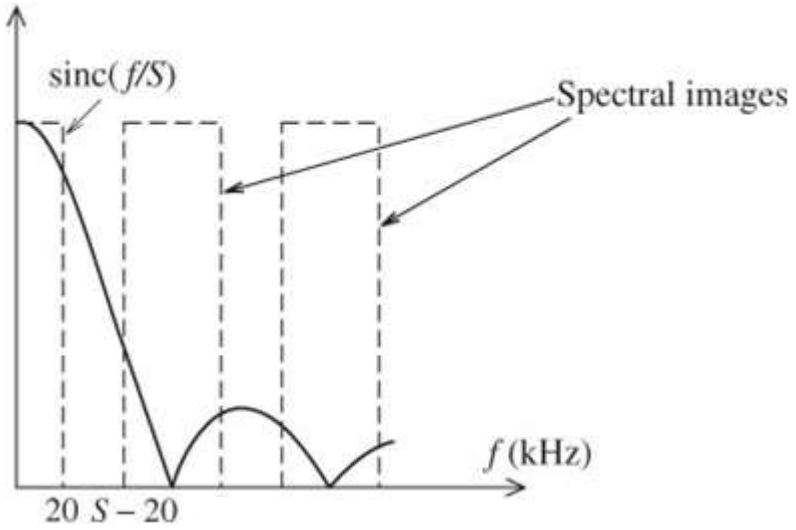


FIGURE 7.29 Staircase reconstruction of a signal at low (left) and high (right) sampling rates. Higher sampling rates produce a much better approximation to the underlying signal and allow the use of a simpler post-filter to smooth out the edges

Rekonstruksjonsfilter

- Digital audio, fs=44.1 kHz
  - Signal mellom 0 og 20 kHz
  - Maks passbåndsdemping: 0.5 dB
  - Stoppbåndsdemping >60 dB, f>S-20=24.1 kHz
- D/A, rett-fram sampling, S=44.1 kHz:
  - S-20 = 44.1-20= 24.1 kHz
  - Transisjonsbånd: 20-24.1 kHz → 80. ordens analogt Butterworth!
- D/A, 4x oversampling, S=176.4 kHz:
  - S-20=176.4-20 = 156.4 kHz
  - Transisjonsbånd: 20-156.4 kHz → 4. ordens Butterworth
    - Enorm reduksjon i kompleksitet
  - Tidlige CD-spillere (4x + 2. eller 3. ordens Bessel ≈ lineær fase)



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Multirate datakonvertering

- Ønsker å erstatte mest mulig analog elektronikk med digital behandling
- ADC er et eksempel, men som nettop vist er bare 40% av fs brukbar båndbredde mot 50% i teorien.
- Dessuten trengs analogt filter etter DAC for interpolasjon

Multirate signalbehandling og Δ-$\sum$

- Fordel å redusere samplingsrate internt i DSP-er
  - Eks: ultralydmottaker for å få færre operasjoner
  - Frekvens 40 kHz, båndbredde 4 kHz, samplet på 17 kHz, desimerte med 2 for å få 8,5 kHz effektiv samplerate
- Fordel å øke samplingsrate før rekonstruksjon
  - Eks: 4 x oversampling i CD-spiller
    - For å få enklere analog filter
    - Enklere kompensasjon for 0-te ordens hold
- Oversampling reduserer behov for bit.
  - Til slutt: trengs bare ett bit

Delta-sigma, Δ-∑

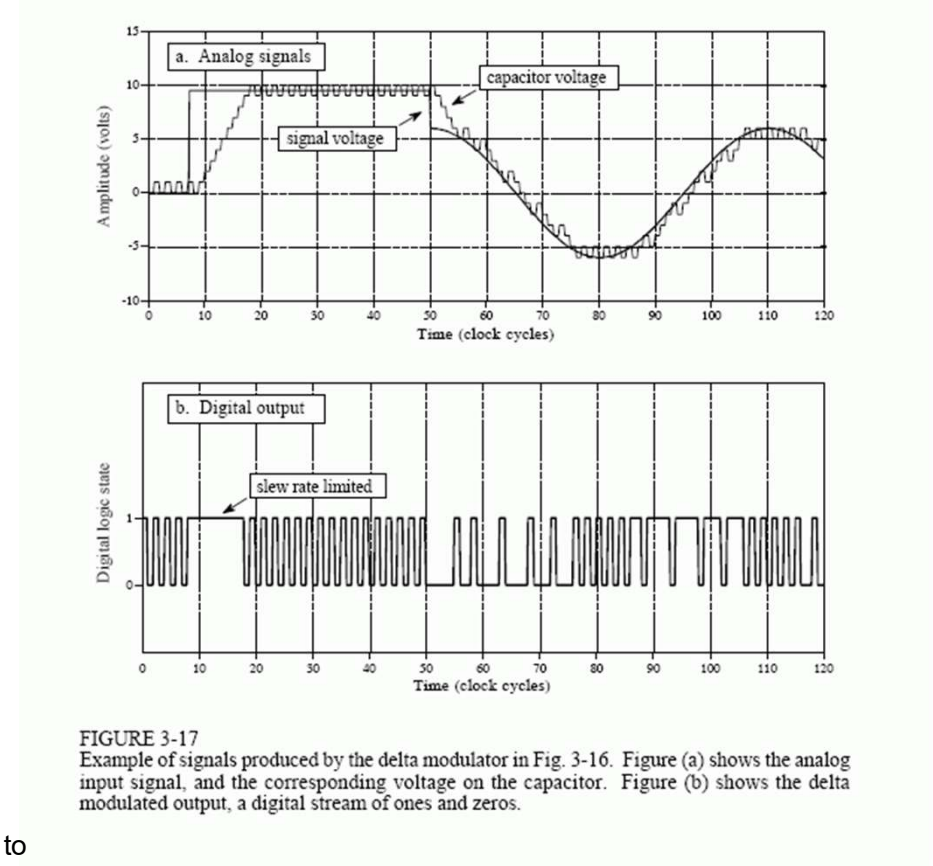
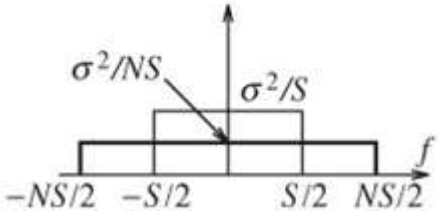


FIGURE 3-17
Example of signals produced by the delta modulator in Fig. 3-16. Figure (a) shows the analog input signal, and the corresponding voltage on the capacitor. Figure (b) shows the delta modulated output, a digital stream of ones and zeros.

Fig: The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith

Kvantisering og oversampling

- Oversampling av analogt signal:
  - Enklere anti-aliasing filter
  - Færre bit i A/D-konverter da kvantiseringsstøy spres over et større frekvensbånd
- Kvantiseringsstøy ~hvit hvis nok bit brukes
  - Hvit ⇔Jevnt spredt over alle frekvenser -S/2<f<S/2
  - Varians $P=\sigma^2=\Delta^2/12$
  - Spektraltetthet: $P_{ee}(f)= \sigma^2/S$

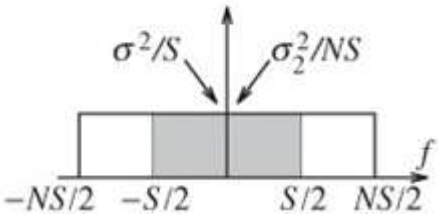


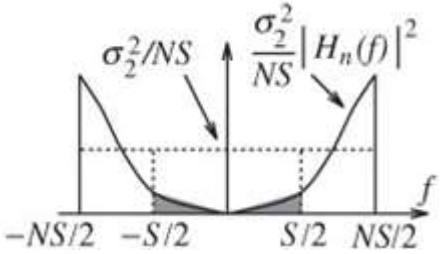
- Ved oversampling med N
  - Spektraltetthet: $P_{ee}(f)= \sigma^2/NS$
  - $10\log N|_{N=2}$ = 3 dB: sparer halv bit pr dobling av samplerate
  - $\Delta B = 0.5\log_2 N$: N=4 -> $\Delta B = 0.5*2 = 1$ bit
  - 1 bits CD? $\Delta B=16-1=0.5\log_2 N$ => $N = 2^{30} \sim 10^9$, NS = 44.1 teraHz!!!

Oversampling og støyforming

- Enda mer gevinst ved forming av støyspektret
- Vanlig støyformer gir sinus-form for |f|<S/2:

$$H_{NS}(f) = |2sin(\pi f)/NS)|^p$$





- p er orden til delta-sigma konverter
- Kan vise besparelse på

  $\Delta B = (p+0.5)\log_2 N - 0.5\log_2(\pi^{2p}/(2p+1))$ bit
- Eks: N=4, p=1: $\Delta B \approx 2.1$ bit
  - Dobbelt så mye som ved ren oversampling
- Eks: N=4, p=3: $\Delta B \approx 3.4$ bit

1-bits sampling (Delta-sigma) i audio

- Må ha besparelse på ΔB=16-1 = 15 bit
  - 1-bits A/D er bare en fortegnsdetektor
- Hva skal til?
  - Oversampling: N = 64, dvs 44.1·16 kHz = 2.8224 MHz
  - Orden: p = 3. ordens støyforming
    - $\Delta B = (0.5+3) \cdot 6 - 3.55 = 17.5$ bit (> 15)
  - Sigma = integrator
  - Delta = differanse
- Praktisk delta-sigma
  - Ill: Wikimedia commons

