Introduction into the memristor and the non-linear electrical properties of human skin

Oliver Pabst, Ørjan Grøttem Martinsen
“If it’s pinched it’s a memristor”


The Chua Lectures: A 12-Part Series with Hewlett Packard Labs
The missing circuit element

The four passive elements [3]

Memristor - the missing circuit element [4]

\[ v = M(Q)i \]


The missing circuit element

The term “memristor” is a combination of “memory” and “resistor”.

Its memristance, \( M(x) \), (in analogy to resistance) is dependent on one or more internal state variables states (expressed by \( x \), a vector of internal state variables). For an ideal memristor, the memristance is a function of the charge, \( Q \). However, \( x \) can be something else like, for example, the extension of a high conductive region vs. a low conductive region.

\[
v = M(x)i
\]

A memristor is described by its state-dependent Ohm’s law

\[
\frac{dx}{dt} = f(x, i) \cdot i
\]

and

the state equation, that describes, how the internal state(s) changes with the current going through the memristor.

Comparison of a resistor and a memristor

Resistor

\[ v = R \cdot i \]

Memristor

\[ \frac{dx}{dt} = f(x,i) \cdot i \]


Different memristor classes

We can also express the memristor in terms of its state-dependent conductance (memductance, in analogy to conductance). Furthermore, different memristor classes are defined and summarized in the table below.

<table>
<thead>
<tr>
<th>Memristor type</th>
<th>Ideal</th>
<th>Ideal generic</th>
<th>Generic</th>
<th>Extended</th>
</tr>
</thead>
<tbody>
<tr>
<td>State dependent Ohms law</td>
<td>$i = G(\phi) \cdot v$</td>
<td>$i = G(x) \cdot v$</td>
<td>$i = G(x) \cdot v$</td>
<td>$i = G(x,v) \cdot v$</td>
</tr>
<tr>
<td>State equation</td>
<td>$\frac{d\phi}{dt} = v$</td>
<td>$\frac{dx}{dt} = g(x) \cdot v$</td>
<td>$\frac{dx}{dt} = g(x,v) \cdot v$</td>
<td>$\frac{dx}{dt} = g(x,v) \cdot v$</td>
</tr>
<tr>
<td>Internal state variable</td>
<td>Flux $\phi$</td>
<td>General variable $x$</td>
<td>Vector of internal state variables $x$</td>
<td></td>
</tr>
<tr>
<td>Indication</td>
<td>*1, *2, *3</td>
<td>*1, *3</td>
<td>*3</td>
<td>*4</td>
</tr>
</tbody>
</table>

*1 Pinched hysteresis loop in the V-I plot is odd-symmetric.
*2 $\varphi$-$q$ plot results in a straight line.
*3 V-I plot tends towards a straight line for $\lim f \to \infty$.
*4 V-I plot tends towards a single valued curve for $\lim f \to \infty$.

Three fingerprints of memristors

-> How to identify memristors?

1. Pinched hysteresis loop in the V-I plot with pinched point in the origin
   -> valid for any signal shape (periodic, non-periodic) and amplitude

2. The lobe area of the pinched hysteresis loop is decreasing with increasing frequency

3. If the frequency tends to infinity, the pinched hysteresis loop should shrink to a single-valued function

Breakthrough in memristor research

LETTERS

The missing memristor found

Dmitri B. Strukov, Gregory S. Snider, Duncan R. Stewart & R. Stanley Williams

Anyone who ever took an electronics laboratory class will be familiar with the fundamental passive circuit elements: the resistor, the capacitor and the inductor. However, in 1971 Leon Chua reasoned from symmetry arguments that there should be a fourth fundamental element, which he called a memristor (short for memory resistor)\(^1\). Although he showed that such an element has many interesting and valuable circuit properties, until now no one has presented either a useful physical model or an example of a memristor. Here we show, using a simple analytical example, that memristance arises naturally in nanoscale systems in which solid-state electronic and ionic transport are coupled under an external bias voltage. These results serve as the foundation for understanding a wide range of hysteretic current–voltage behaviour observed in many nanoscale electronic devices\(^2\) that involve the motion of charged atomic or molecular species, in particular certain titanium dioxide cross-point switches\(^3\).---

propose a physical model that satisfies these simple equations. In 1976 Chua and Kang generalized the memristor concept to a much broader class of nonlinear dynamical systems they called memristive systems\(^4\), described by the equations

\[ \begin{align*}
\tau &= R(w, i) i \\
\frac{dw}{dt} &= f(w, i)
\end{align*} \]

where \( w \) can be a set of state variables and \( R \) and \( f \) can in general be explicit functions of time. Here, for simplicity, we restrict the discussion to current-controlled, time-invariant, one-port devices. Note that, unlike in a memristor, the flux in memristive systems is no longer uniquely defined by the charge. However, equation (3) does serve to distinguish a memristive system from an arbitrary dynamical device, no current flows through the memristive system when the


The Hewlett Packard (HP) memristor model

\[ M(x) = 0.1 \cdot x + 16 \cdot (1 - x) \] [kΩ]

\[ \frac{dx}{dt} = \text{const} \cdot i \] [8]

The memristance in this example is a function of \( x \), the extension of the doped titandioxide (TiO\(_{2-x}\)) region (high conductive) vs. the non-doped titandioxide (TiO\(_2\)) region (low conductive).

The red bar illustrates the extension, \( x \), of the doped region. The results are obtained by simulation (using a sinusoidal voltage source).

With a positive voltage, \( x \) increases and the memristance, \( M(x) \), decreases consequently (see first quadrant). \( \rightarrow \) The current «on the way back» is larger than on the «way up».
With a negative voltage, \( x \) decreases and the memristance increases consequently. The resulting current decreases consequently.


**Examples of pinched hysteresis loops**

![Pinched Hysteresis Loops of Sodium Ion Channel Memristor](image)


Closure theorem of memristors
Is this a memristor?

Yes:
This plot is obtained by simulation (based on an adapted version of the HP memristor).
The two branches of the loop are crossing the pinched point with different slopes. We call this type of memristor a “transversal” memristor.
Is this a memristor?

Yes,
this plot is obtained by simulation of a memristor model presented in [11].
The two branches of the loop do not need to cross the pinched point but can also touch the pinched point with equal slopes. We call this type of memristor a “tangential” memristor.

Is this a memristor?

Yes,

This is plot is obtained from some solid state memristor presented in [12].

Is this a memristor?

This is not a memristor, since we do not obtain pinched hysteresis loops for applied sinusoidal voltage with amplitudes, $A$, of 1.5 V and 2 V.

Is this a memristor?

This is also not a memristor, since we do not obtain pinched hysteresis loops for applied sinusoidal voltage with amplitudes, $A$, of 1.5 V and 2 V.

Yes,
This is a memristive model of human skin as presented in [13]. The sweat is moved by electro-osmosis resulting in a change of the state dependent conductance.

Is this a memristor?

Yes, this is a recording from the human skin memristor (sinusoidal voltage with excitation frequency of 0.25 Hz). However, measurements on human skin and other biological tissues (like apples) are affected by parasitic elements like, a capacitance. That is why the pinched point is slightly shifted from the origin of coordinates.

If you want to design an experiment to find out whether a material or tissue is a memristor, what do you need to test?

-> Remember what the three fingerprints of a memristor are.
Non-linear electrical properties of human skin
**Instrumentation**

[Image of a diagram showing electrode placement and skin layers: Epidermis, Dermis, Low conductance, High conductance]


Non-linear AC characteristics

Different signal frequencies

Non-linear AC characteristics

Different amplitudes

Non-linear AC characteristics

Non-linear AC characteristics

Different subjects

Parameterization

How to evaluate the recordings of several subjects?

First idea is to use the geometry of the pinched hysteresis loop (see plot on the left). However, what are the parameters that are meaningful?
Non-linearity (adapted from [14])

Evaluation over several test subjects

[Image of box plots showing non-linearity (NL) across different frequencies and voltages, with a linear trend line labeled as >95% non-linear and a separate linear trend line.]

Evaluation over several test subjects

Lobe area

Maximum current

Outlook and Motivation

Undeveloped research field

New insights

Sensor applications

Diagnostics?
Sensorama 2018
Data acquisition box
Picoscope

**Advantage**
- easy to set up

**Disadvantages**
- less automation
- output voltage limited to ±2
Electrodes in saline solution

\[ f = 0.001 \text{Hz} \]
\[ \text{amp} = 0.1 \, \text{V} \]
Electrodes in saline solution

\[ f = 0.025 \text{Hz} \]
\[ \text{amp} = 0.1 \text{ V} \]
Measuring on plants with a three-electrode system

Design of the studies

- Different frequencies
- Different amplitudes
- Different excitation signals

- DC pulses?
Design of the studies

- Duration of the experiments: Think of excitation frequency of 0.001 Hz

- Need of a good experiment protocol
„Where there is ion, there is memristor.“ [11]
If you are interested in learning more about memristors, the Chua lectures (1 to 5 out of 12) are quite useful. You can find them on YouTube.