FYS 4130 Statistical Mechanics

Homework 7 March 4, 2011

1) Black body radiation

Consider a cavity of volume V containing black body radiation. The energy of a photon is a function of the frequency $\epsilon = \hbar\omega$. This is grand canonical ensemble of photons which does not have a fixed number of particles. The chemical potential for photons is $\mu = 0$.

a) We can approximate the sum over momentum states as an integral over the frequencies $d\omega$. Calculate the density of states $D(\omega)$ for a massless, relativistic particle with $E = pc = \hbar\omega$. You need to include a factor of 2 for the two polarization states of the photons.

b) Calculate the grand canonical partition function. Here it is easier to calculate $\ln \Xi$. The result is a sum over frequencies ω_i

$$\ln \Xi = \sum_i \ln\left(\frac{1}{1 - e^{-\beta\hbar\omega_i}}\right)$$

b) Use the grand canonical partition function to find the energy density of the black body radiation.

c) Find the pressure and the relation betwen pressure and energy density.

d) Find the entropy density for the photons.

2) Neutrino background of the universe.

In the early universe, the matter and photons were in thermal equilibrium. As the temperature fell with expansion to below $T \sim 10^{11} K$, the neutrinos decoupled from the other particles. In this regime the neutrinos can be treated as noninteracting, massless, relativistic particles.

a) Calculate the number density of the neutrino background of the universe.

b) Calculate the entropy density for the neutrino background.

3) Neutrino background for massive neutrinos

Previously, to calculate the properties of the neutrino background of the universe, we assumed the neutrinos were noninteracting, relativistic, massless particles. The mass of the neutrino is small, but nonzero. Consider the neutrino background as a gas of noninteracting, relativistic, particles with mass.

a) Calculate the density of states for the massive neutrinos. Use the relativistic relation for momentum and energy: $E^2 = P^2c^2 + m^2c^4$.

b) Assuming that the mass of the neutrino is small, keep only the first order terms in the mass of the neutrino. Calculate the number density and energy density of the neutrino background.

c) Assume the neutrino mass is $m_\nu \sim 10^{-3} eV$, at what temperature does the correction for the mass of the neutrino become important?