UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam: FYS4130 statistical mechanics.

Day: Monday, June 8, 2009.

Time: 14:30 - 17:30

Problem set consists of: 2 pages.

Attachments: None.

Approved aids: Personal lecture notes, compendium, approved mathematical tables and electronic calculator.

Check that the problem set is complete before you start answering the questions.

Problem 1

A gas of free bosons in three dimensions is in thermal equilibrium with temperature T. The energy of a particle with momentum $\mathbf{p}$ has the unusual form $E = A|\mathbf{p}|^{1/2}$ where A is a constant.

a) Calculate the chemical potential of the gas in the classical limit as a function of density and temperature.

b) What is the internal energy per particle in the gas?

c) Quantum statistics must be used when the temperature is reduced. Show that the gas will then have a transition to a condensed phase and find the corresponding critical temperature T_c as a function of the density.

d) What fraction of the total number of particles in the gas has condensed when the temperature is $T = T_c/2$?

Problem 2

Diffusion of a local particle density $C(x,t)$ in one dimension is governed by the equation

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

where D is the diffusion constant.

a) Show that

$$C(x,t) = \sqrt{\frac{1}{4\pi Dt}}e^{-x^2/4Dt}$$

is a solution of the diffusion equation. What is special with this solution?

b) At time $t = 0$ the concentration is given as $C_0(x) = Ke^{-x^2/L^2}$ where L is a fixed length. Find the constant K from the requirement that there are in total N diffusing particles.

c) What is the current of particles for $x = L$ at this time?

d) Calculate the concentration of particles at a later time $t > 0$.

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In the first problem you can make use of the following value of the Riemann zeta-function:

$$\zeta_R(6) = \frac{\pi^6}{945}$$