

Problem set 11 for the course FYS4130

April 9, 2013

Random walks in grade space

taken from **Statistical Mechanics: Entropy, Order parameter, and Complexity** by **James P. Sethna, Oxford University Press 2006**. Let us make a model of the grade distribution in an exam. Let us imagine a multiple-choice test of ten problems of ten points each. Each problem is identically difficult, and the mean is 70. How much of the point spread on the exam is just luck, and how much reflects the differences in skill and knowledge of the people taking the exam? To test this, let us imagine that all students are identical, and that each question is answered at random with a probability 0.7 of getting it right.

(a) *What is the expected mean and standard deviation for the exam? (Work it out for one question, and then use our theorems for a random walk with ten steps.)*

A typical exam with a mean of 70 might have a standard deviation of about 15.

(b) *What physical interpretation do you make of the ratio of the random standard deviation and the observed one?*

Thermal diffusion

taken from **Statistical Mechanics: Entropy, Order parameter, and Complexity** by **James P. Sethna, Oxford University Press 2006**. The rate of energy flow in a material with thermal conductivity k_t and a temperature field $T(x, y, z, t) = T(r, t)$ is $\mathbf{J} = k_t \nabla T$. Energy is locally conserved, so the energy density E satisfies $\partial E / \partial t = \nabla \cdot \mathbf{J}$.

(a) *If the material has constant specific heat c_p and density ρ , so $E = c_p \rho T$, show that the temperature T satisfies the diffusion equation $\partial T / \partial t = k_t / (c_p \rho) \nabla^2 T$.*

(b) *By putting our material in a cavity with microwave standing waves, we heat it with a periodic modulation $T = \sin(kx)$ at $t = 0$, at which time the microwaves are turned off. Show that the amplitude of the temperature modulation decays exponentially in time. How does the amplitude decay rate depend on wavelength $\lambda = 2\pi/k$?*