Problem set 12 for the course FYS4130

April 19, 2013

Discrete random walk

Let us consider a discrete random walk with a continuous jump size distribution. We will denote the position of the walker at a discrete time N by x_N. It is obtained as a sum of independent jumps

$$x_N = \sum_{n=0}^{N} \xi_n \tag{1}$$

where the size of the jumps ξ_n is assumed to be statistically independent and identically distributed uniformly in the interval between -1 and 1, i.e. $\xi = U(-1,1)$. We also have that $x_0 = 0$.

(a) What is the characteristic function of the uniform jump probability distribution function $p_\xi(\xi)$?

(b) Use the convolution theorem of probabilities to calculate the probability distribution function (pdf) for x_N for $N = 2$? We will denote this probability by $p(x, N = 2)$.

(c) Calculate $p(x, N = 2)$ using the characteristic function from question (a).

(d) Obtain the characteristic function, $G_x(k, N)$, valid for any time N, and show how it relates to $p(x, N)$.

Diffusion coefficient and random walk

Consider a discrete-time random walk a jump size that can take values $\pm\Delta x$ with equal probablity. The position of the walker at a discrete time $t = N\Delta t$ depends on the previous position by

$$x_t = x_{t-\Delta t} + \xi_t, \tag{2}$$

where $\langle \xi_t \rangle = 0$ and $\langle \xi_t^2 \rangle = \Delta x^2$.

(a) Show that the standard deviation of the position $\sigma^2 = \langle x_t^2 \rangle - \langle x_t \rangle^2$ can be written as $\sigma^2 = 2Dt$. Find the diffusion coefficient D as a function of Δx and Δt.

(b) Show that in the limit of $\Delta x, \Delta t \to 0$, the evolution equation for the position reduced to overdamped limit of the Langevin equation.