Problem set 7 for the course FYS4130

February 27, 2013

7.10

from Yuri's exercise book

Phonons on a string

taken from Statistical Mechanics: Entropy, Order parameter, and Complexity by James P. Sethna, Oxford University Press 2006. A continuum string of length L with mass per unit length μ under tension τ has a vertical, transverse displacement $u(x,t)$. The kinetic energy density is $(\mu/2)(\partial u/\partial t)^2$ and the potential energy density is $(\tau/2)(\partial u/\partial x)^2$. The string has fixed boundary conditions at $x = 0$ and $x = L$.

(a) Write the kinetic energy and the potential energy in new variables, changing from $u(x,t)$ to normal modes $q_k(t)$ with $u(x,t) = \sum_n q_{k_n}(t)\sin(k_n x)$, where $k_n = n\pi/L$.

(b) Show in these variables that the system is a sum of decoupled harmonic oscillators.

(c) Calculate the density of normal modes per unit frequency $g(\omega)$ for a long string L.

(d) Calculate the specific heat of the string $c(T)$ per unit length in the limit $L \to \infty$, treating the oscillators quantum mechanically. What is the specific heat of the classical string? (Hint: convert the sum into integral for $L \to \infty$).