UNIVERSITY OF OSLO

Department of physics

Compulsory assignment: FYS4130 Statistical physics

Date: Hand in by Mars 26 2015

All symbols that are not explicitly defined are understood to have the definition given in the Lecture notes. The set contains two pages.

Part 1 We will study the distribution of a variable $X = \sum_{i=1}^{N} x_i$ where the random increments x_i are distributed according to different distributions. Such a sum is called a random walk and may be taken as a model for the motion of a molecule in a gas or a liquid.

We will set $x_0 = 1$ and update $x_{i+1} = -x_i$ with probability p and $x_{i+1} = x_i$ with probability $1 - p$. This will introduce a positive correlation over time in the velocity of the particle if $p < 1/2$.

1. Calculate $\langle X^2 \rangle$ for both correlated and uncorrelated velocities by setting $p = 1/2$, $p = 1\%$ and $p = 10\%$ Calculate the ensemble average with 10^5 realizations and use $N = 1000$
2. Make a logarithmic plot, i.e. plot the variance $\log_{10}(\langle \Delta X^2 \rangle)$ as a function of $\log_{10}(t)$ with $t = i$ and identify linear portions of the graphs. Discuss the corresponding early time exponents of the $p = 1\%$ and 10% graphs and why they change at later times.
3. For $p = 0.5$, show that we may write $\langle \Delta X^2 \rangle = 2Dt$ and identify the value of D.
4. If $X(t)$ is taken to be a description of the position of a molecule in a gas, then give a physical interpretation of the p-variations. In particular, what state of the gas would small p values correspond to?
5. Make a histogram of the X_N-values to obtain the distribution $P(X)$ for $p = 0.5$ and 0.9. Plot $\ln(P(X))$ as a function of X^2 and use this to write down an analytic expression for $P(x)$. Discuss the difference for the different p-values
6. Use the analytic expression of $P(x)$ to calculate $\langle X^2(t) \rangle = \int dx x^2 P(x) / \int dx P(x)$. Use this to express a in terms of D in the $p = 0.5$ case. Obtain a from the histogram and compare the values of D.

Part 2

We shall study the black body radiation field in a fictituous 2 dimensional world. The expression in equation (5.38) in the text book is still valid in this case.

1. Assuming periodic boundary conditions, how must the wave numbers k_x and k_y be quantized ?

2. Calculate the internal energy of the field $U(T)$ and the corresponding heat capacity.