Answers to numerical part

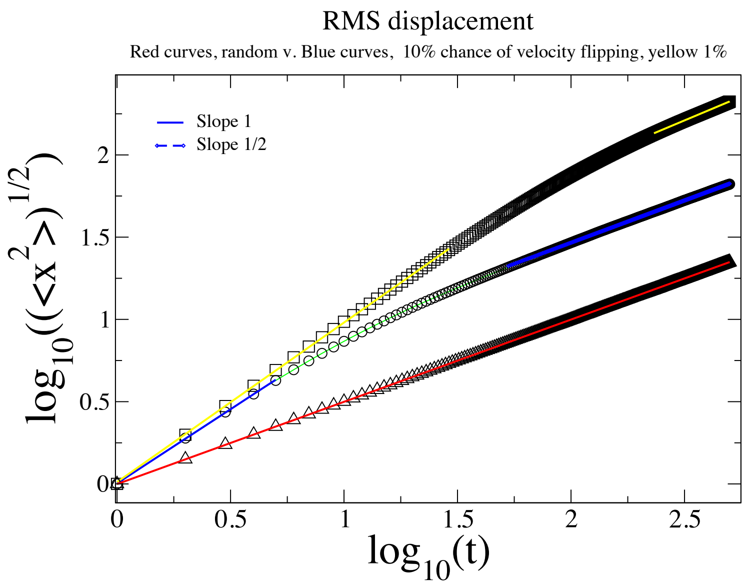

Figure 1: The average (rms) displacement as a function of time Here $N = 500$ and the ensemble contains 100000 runs. .

In figure 1 $\langle X^2(t)\rangle$ is shown for the three cases $p = 0.5, 0.1$ and 0.01. When $p = 0.5$ there are no correlations in x_i's and $\langle X^2(t)\rangle \sim \sqrt{t}$. When $p < 0.5$ the positive correlations imply that for a typical time $t \sim 1/p$ the direction of the velocity (or sign of x_i) is constant. During this correlation time $\sqrt{\langle X^2(t)\rangle} \sim t$ while later on $\sqrt{\langle X^2(t)\rangle} \sim \sqrt{t}$. The small p-values give large mean-free-paths, which physically corresponds to a dilute gas.

In figure 2 we see that $\ln P(X) = -aX^2$, or equivalently that $P(X)$ is a Gaussian:

$$P(x) \sim e^{-aX^2}. \tag{1}$$

The slope of the $p = 0.5$ part of the figure gives $a = 0.000503$, and

$$\langle X^2\rangle = \frac{\int dx x^2 e^{-ax^2}}{\int dx e^{-ax^2}} = -\frac{\partial}{\partial a}\ln\int dx e^{-ax^2} = \frac{1}{2a} = 2Dt \tag{2}$$

so that $a = 1/(4Dt)$. With $t = 1000$ this gives $D = 1/(4at) = 0.495$ in reasonable agreement with the result of figure 1 which gives $\log_{10}(2D) = 0$ or $D \approx 0.5$.

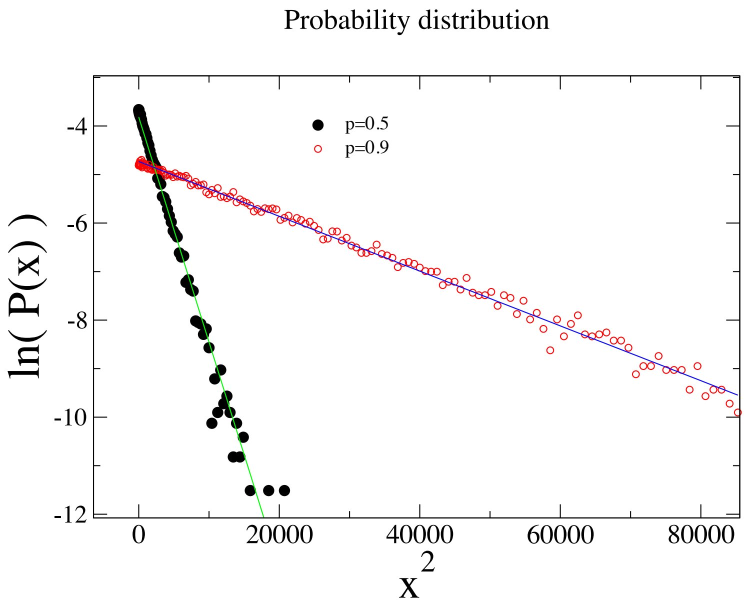

Figure 2: The distribution of displacements.