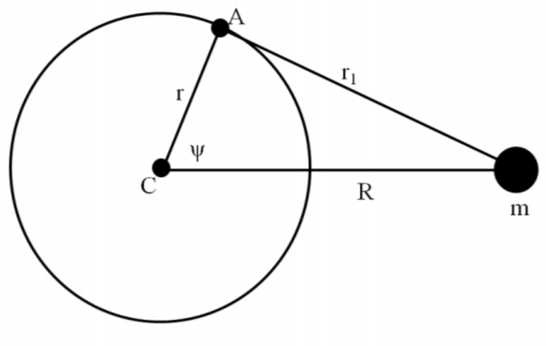
**Lecure 3 220118**

**The tidal field on the Earth due to the Moon**




We have the following equation:

$$r_1^2 = R^2 - 2rR\cos\psi + r^2$$

We calculate $1/r_1$ in order to calculate the tidal potential as

$$r_1^{-1} = \frac{1}{R}\left[1 - 2\frac{r}{R}\cos\psi + \left(\frac{r}{R}\right)^2\right]^{-1/2} \qquad (4.2)$$

Assuming $r$ is sufficiently smaller than $R$, we have

$$r_1^{-1} = \frac{1}{R}\left[1 + \frac{r}{R}\cos\psi + \frac{1}{2}\left(\frac{r}{R}\right)^2\left(3\cos^2\psi - 1\right)\right] \quad (4.3)$$

Where we have applied the Taylor expansion (1.25B) to 2. Order in *p*. The potential at a point A on the surface of the Earth in the gravitational field of the Moon is

$$V_m = -\frac{Gm}{r_1} = -\frac{Gm}{R}\left[1 + \frac{r}{R}\cos\psi + \frac{1}{2}\left(\frac{r}{R}\right)^2\left(3\cos^2\psi - 1\right)\right] \quad (4.4)$$

The first term, $V_C = -\dfrac{Gm}{R}$, is the potential in the gravitational field of the Moon at the center, C, of the Earth.

The second term, $V_A = -\dfrac{Gm}{R^2}r\cos\psi = -g_{Moon}x_A$, is the difference of the potential at C and A in the gravitational field of the Moon if one neglects the inhomogeneity in the Moon's field at the Earth. Hence the sum of the first two terms is then the potential at A in the gravitational field of the Moon.

This means that the third term,

$$V_t = -\frac{Gm}{2R^3} r^2 \left(3\cos^2\psi - 1\right),$$

is the difference between the potential at A in the Moon's gravitational field if the field is considered homogeneous with the value at the center of the Earth, and the actual potential at A. This difference is due to the inhomogeneity of the gravitational field of the Moon at the Earth, i. e. it is due to the tidal gravitational field. It is therefore called *the tidal potential* at A.

The height difference, $\Delta h$, between flood and ebb due to the Moon's tidal field is given by

$$g\Delta h = V_t(0) - V_t(\pi/2)$$

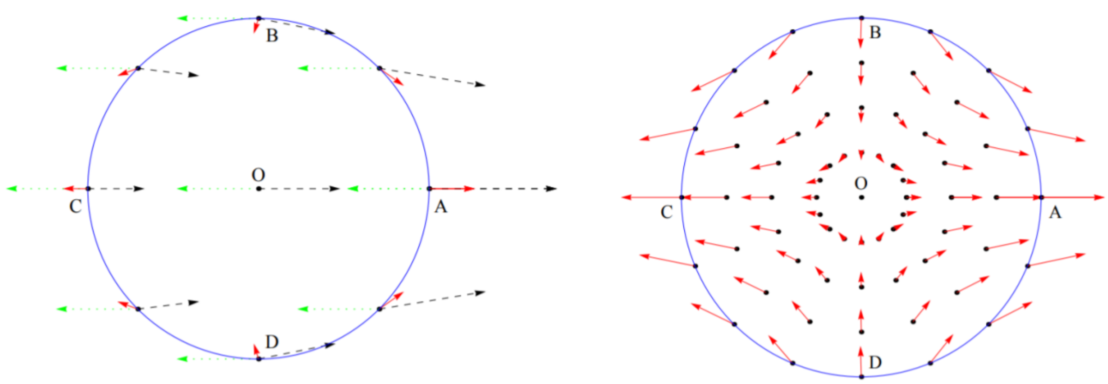
where $V_t = V_t(\psi)$, and g is the acceleration of gravity at the surface of the Earth. This gives

$$\Delta h = \frac{3}{2}\frac{Gm}{g}\frac{r^2}{R^3}.$$

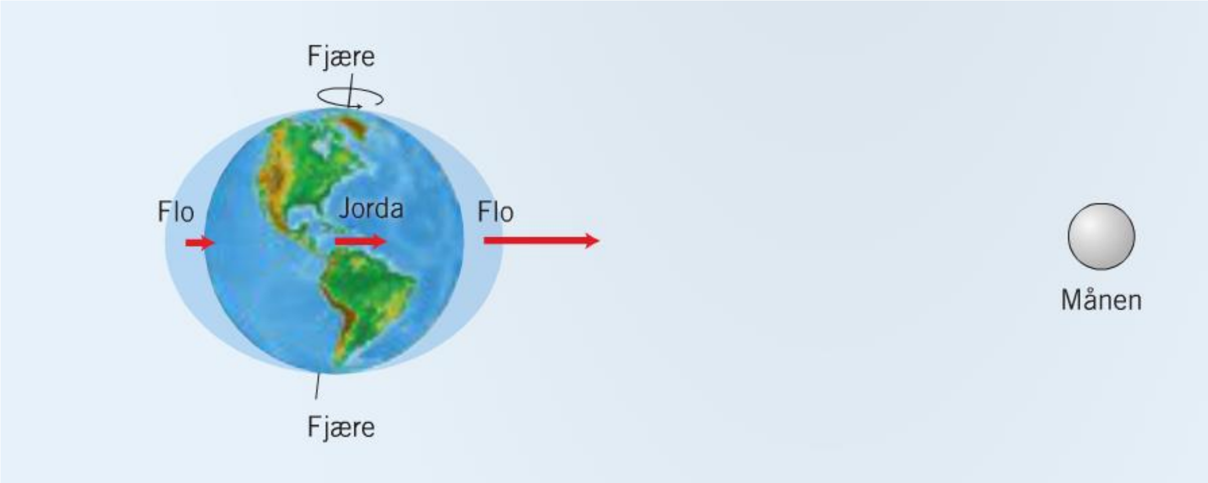
For a numerical result we need the following values:

$$M_{\text{Moon}} = 7.35 \cdot 10^{25}\text{g}, \qquad g = 9.81\text{m/s}^2, \tag{1.33}$$
$$R = 3.85 \cdot 10^5\text{km}, \qquad r_{\text{Earth}} = 6378\text{km}. \tag{1.34}$$

With these values we find $\Delta h = 53\text{cm}$, which is typical of tidal height differences.




The *tidal acceleration field* (red) at the surface of the Earth due to the Moon is the acceleration of gravity at the surface (black) of the Earth minus the acceleration of gravity at the center (green) of the Earth in the Moon's gravitational field.




## 1.4 The Principle of Equivalence

Galilei investigated experimentally the motion of freely falling bodies. He found that they moved in the same way, regardless what sort of material they consisted of and what mass they had.

In Newton's theory of gravitation mass appears in two different ways; as gravitational mass, $m_G$, in the law of gravitation, analogously to charge in Coulomb's law, and as inertial mass, $m_I$ in Newton's 2nd law.

The equation of motion of a freely falling particle in the field of gravity from a spherical body with mass $M$ then takes the form

$$\frac{d^2\vec{r}}{dt^2} = -G\frac{m_G}{m_I}\frac{M}{r^3}\vec{r}. \tag{1.35}$$

The results of Galilei's measurements imply that the quotient between gravitational and inertial mass must be the same for all bodies. With a suitable choice of units, we then obtain

$$m_G = m_I. \tag{1.36}$$

Measurements performed by the Hungarian baron Eötvös around the turn of the century indicated that this equality holds with an accuracy better than $10^{-8}$. More recent experiments have given the result $|\frac{m_I}{m_G} - 1| < 9 \cdot 10^{-13}$.

Einstein assumed the general validity of Eq.(1.36) and considered it a consequence of a principle which he called *the principle of equivalence* and which became one of the foundational principles of the general theory of relativity.

A consequence of this principle is the possibility of removing the effect of a gravitational force by being in free fall. In order to clarify this, Einstein considered a homogeneous gravitational field in which the acceleration of gravity, g, is independent of the position. In a freely falling, non-rotating reference frame in this field, all free particles move according to

$$m_I \frac{d^2\vec{r}}{dt^2} = (m_G - m_I)\vec{g} = 0, \tag{1.37}$$

where eq. (1.36) has been used.

This means that an observer in such a freely falling reference frame will say that the particles around him are not acted upon by forces. They move with constant velocities along straight paths. In other words, such a reference frame is inertial.

Einstein's heuristic reasoning suggests equivalence between inertial frames in regions far from mass distributions, where there are no gravitational fields, and inertial frames falling freely in a gravitational field. This equivalence between all types of inertial frames is so intimately connected with the equivalence between gravitational and inertial mass, that the term "principle of equivalence" is used whether one talks about masses or inertial frames. The equivalence of different types of inertial frames encompasses all types of physical phenomena, not only particles in free fall.

The principle of equivalence has also been formulated in an "opposite" way. An observer at rest in a homogeneous gravitational field, and an observer in an accelerated reference frame in a region far from any mass distributions, will obtain identical results when they perform similar experiments. An inertial field caused by the acceleration of the reference frame, is equivalent to a field of gravity caused by a mass distribution, as far is tidal effects can be ignored.

In the general theory of relativity gravitation is not considered to be a force. Instead one talks about gravitational acceleration. A gravitational field is an acceleration field, meaning that there is an acceleration of gravity at every point in a gravitational field.

It is important to designate between local and non-local aspects of a gravitational field. Here "local" means that the extension in space and time is so small that tidal effects cannot be measured. In this sense *a gravitational field is locally homogeneous* corresponding to the statement that spacetime is locally flat.

Tidal effects are due to the inhomogeneity in a gravitational field. In this sense it represents a non-local effect. One can show that *tidal effects in Newton's theory correspond to spacetime curvature in Einstein's theory.*

Experiencing acceleration of gravity has a local character. It has nothing to do with spacetime curvature. In general one experiences acceleration of gravity when one is not in an inertial frame, i.e. when one is not in a freely falling, nonrotating frame.