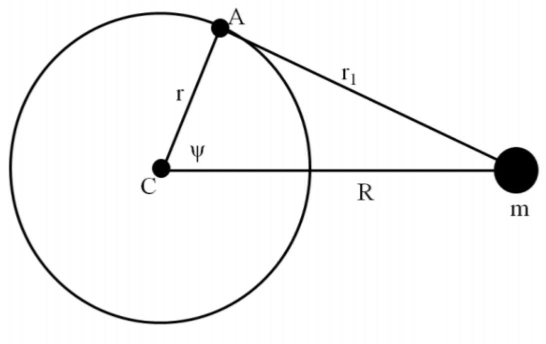
Lecure 3 220118

The tidal field on the Earth due to the Moon



We have the following equation:

$$r_1^2 = R^2 - 2rR\cos\psi + r^2$$

We calculate $1/r_1$ in order to calculate the tidal potential as

$$r_1^{-1} = \frac{1}{R}\left[1 - 2\frac{r}{R}\cos\psi + \left(\frac{r}{R}\right)^2\right]^{-1/2} \qquad (4.2)$$

Assuming r is sufficiently smaller than R, we have

$$r_1^{-1} = \frac{1}{R}\left[1 + \frac{r}{R}\cos\psi + \frac{1}{2}\left(\frac{r}{R}\right)^2\left(3\cos^2\psi - 1\right)\right] \quad (4.3)$$

Where we have applied the Taylor expansion (1.25B) to 2. Order in *p*. The potential at a point A on the surface of the Earth in the gravitational field of the Moon is

$$V_m = -\frac{Gm}{r_1} = -\frac{Gm}{R}\left[1 + \frac{r}{R}\cos\psi + \frac{1}{2}\left(\frac{r}{R}\right)^2\left(3\cos^2\psi - 1\right)\right] \quad (4.4)$$

The first term, $V_C = -\frac{Gm}{R}$, is the potential in the gravitational field of the Moon at the center, C, of the Earth.

The second term, $V_A = -\frac{Gm}{R^2} r\cos\psi = -g_{Moon}x_A$, is the difference of the potential at C and A in the gravitational field of the Moon if one neglects the inhomogeneity in the Moon's field at the Earth. Hence the sum of the first two terms is then the potential at A in the gravitational field of the Moon.

This means that the third term,

$$V_t = -\frac{Gm}{2R^3} r^2 \left(3\cos^2\psi - 1\right),$$

is the difference between the potential at A in the Moon's gravitational field if the field is considered homogeneous with the value at the center of the Earth, and the actual potential at A. This difference is due to the inhomogeneity of the gravitational field of the Moon at the Earth, i. e. it is due to the tidal gravitational field. It is therefore called *the tidal potential* at A.

The height difference, Δh, between flood and ebb due to the Moon's tidal field is given by

$$g\Delta h = V_t(0) - V_t(\pi/2)$$

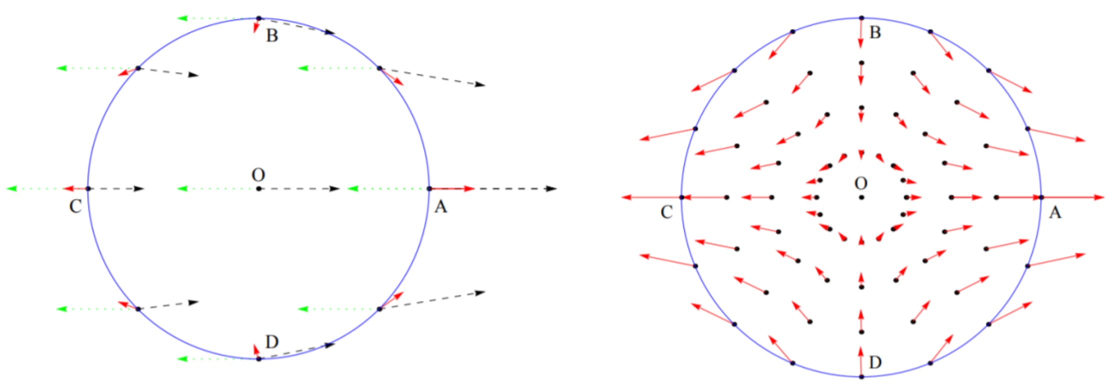
where $V_t = V_t(\psi)$, and g is the acceleration of gravity at the surface of the Earth. This gives

$$\Delta h = \frac{3}{2}\frac{Gm}{g}\frac{r^2}{R^3}.$$

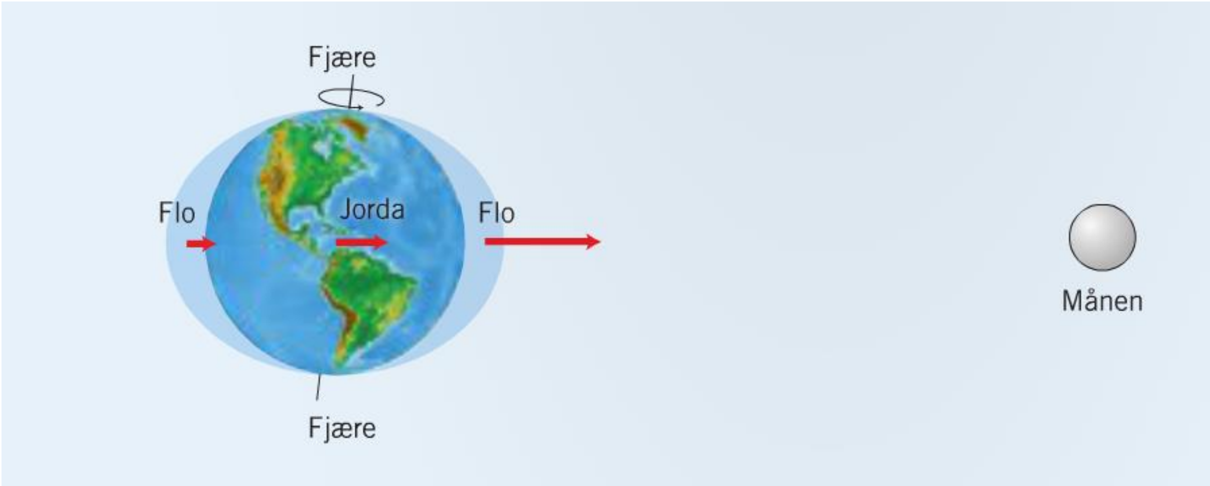
For a numerical result we need the following values:

$$M_{\text{Moon}} = 7.35 \cdot 10^{25}\text{g}, \qquad g = 9.81\text{m/s}^2, \tag{1.33}$$
$$R = 3.85 \cdot 10^5\text{km}, \qquad r_{\text{Earth}} = 6378\text{km}. \tag{1.34}$$

With these values we find $\Delta h = 53\text{cm}$, which is typical of tidal height differences.



The *tidal acceleration field* (red) at the surface of the Earth due to the Moon is the acceleration of gravity at the surface (black) of the Earth minus the acceleration of gravity at the center (green) of the Earth in the Moon's gravitational field.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