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http://www.uio.no/studier/emner/matnat/fys/FYS4160/v22/

Lecture spring 2022:

General Relativity

Problem sheet 0

(Does not count towards the 10% 'bonus points' achievable for the exam)

⇝ These problems are scheduled for discussion on **Tuesday, 25 January 2022**, along with discussing questions related to necessary background knowledge for this course ('FYS4160: preliminaries').

Problem 1

This problem serves as a reminder to practice the use of index notation.

a) Write the following in index notation:

- ∇S (where S is a scalar).
- $\nabla \cdot \mathbf{A}$, $\nabla \times \mathbf{A}$ (where $\mathbf{A}$ is a 3D vector).
- Trace and Transpose of a matrix M.

b) Prove the following 3D identities, using index notation:

- $\nabla \cdot (\nabla \times \mathbf{A}) = 0$, where $\mathbf{A}$ is a 3D vector.
- $\nabla \times (\nabla S) = 0$, where S is a scalar.

Problem 2

And another fresh-up...

a) Are these equalities valid? Correct where necessary!

- $\partial_\mu x^\nu = \delta^\nu_\mu$
- $\partial_\mu x^\mu = 1$
- $\partial^\mu x^\nu = g^{\mu\nu}$
- $T_\alpha{}^\beta{}_\gamma = g^{\beta\mu} T_{\alpha\mu\gamma} = g^{\mu\beta} T_{\alpha\mu\gamma}$
- $T_\alpha{}^\beta{}_\beta = g_{\alpha\mu} g^{\beta\alpha} T^\mu{}_{\alpha\beta}$

b) Construct

- all independent Lorentz scalars from two four-vectors A and B
- all independent Lorentz scalars from (up to two copies of) a rank-2 tensor T, as well as from invoking one rank-2 tensor T and two four-vectors A and B
- all independent Lorentz four-vectors from a scalar S and two four-vectors A and B

Problem 3

In special relativity, time passes slower in a moving frame than in the rest frame:

a) Consider a clock that moves with speed v with respect to an observer. Derive the 'time-dilation factor' between the time that the moving clock shows and the time that the observer measures in its rest frame, using *i)* a spacetime diagram that includes the worldlines of the two observers and *ii)* the explicit form of a Lorentz boost as derived in the lecture.

b) Most of the high-energy secondary cosmic rays that reach the earth are muons, which decay with a lifetime of $\tau_0 = 2.2 \times 10^{-6}\,\mathrm{s}$ into electrons and (anti-) neutrinos, $\mu^\pm \to e^\pm + \nu_\mu + \bar{\nu}_e$. Assume that the muons are produced at an altitude of $h = 10\,\mathrm{km}$ and move downwards, perpendicular to the surface of the earth, with an energy of $E = 1\,\mathrm{GeV}$. Given a mass of $m_\mu = 106\,\mathrm{MeV}$, what does that imply for their velocity in terms of c? Discuss the flux ratio Φ_h/Φ_0, where Φ_h is the flux at $h = 10\,\mathrm{km}$ and Φ_0 the flux at ground, both in the muon rest frame and in the laboratory system (i.e. the surface of the earth)!
[Hint: The conclusion should be that the concepts of 'length contraction' and 'time dilation' are indeed the same thing, seen from different perspectives. Reminder: flux is the product of number density and velocity, and quantum mechanical decay is always exponential.]
What would be the result of a non-relativistic calculation?