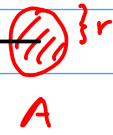
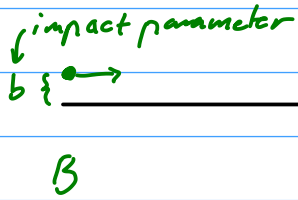


Scattering cross section and decay rate

Cross section $\sigma \sim$ effective target area seen by an interacting particle.

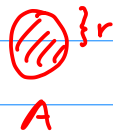


Hard sphere approximation:

Scattering happens (only) when $b < r$.

$$\Rightarrow \sigma = \pi r^2$$

Consider many incident particles (number density n_B):

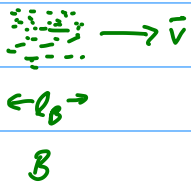


$$\# \text{events during } dt = n_B \cdot \sigma \cdot v dt \quad (*)$$

In general: $\sigma \stackrel{\text{def}}{=} \frac{\# \text{events during } dt / dt}{v \cdot n_B} \left(= A \frac{\# \text{events during } dt}{A v dt n_B} = A \frac{\# \text{events d.t.}}{N_B} \right)$

define σ by (*) such that it becomes the effective area of the target.

Consider two colliding beams:



$$\# \text{events} < l_A l_B n_A n_B A$$

$\hookrightarrow$ overlapping area

Define $\sigma = \frac{\# \text{events}}{n_A n_B l_A l_B A} = A \frac{\# \text{events}}{n_A A l_A n_B A l_B} = A \frac{\# \text{events}}{N_A N_B}$

$\uparrow$ total # particles A and B

Decay rate: $\Gamma = \frac{\# \text{decays/time}}{\# \text{particles still left}} = - \frac{dn/dt}{n} \Rightarrow n = n_0 e^{-\Gamma t}$

$\hookrightarrow n$ for $t=0$.

Life time $\tau = \frac{1}{\Gamma}$.

Obtaining σ from $\mathcal{M}$

Derivation in PS.

only part of cross section due to scattering into $\pi d^3 p_f$ about $\{p_f\}$.

4-mom. in

n particles

$$d\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|} d\pi_n |\mathcal{M}(p_A p_B \rightarrow \{p_f\})|^2$$

$\underbrace{\hspace{10em}}_{\text{E velocities.}}$

$\mathcal{M}$ "inv. matrix element", interesting (model-dep.) part.

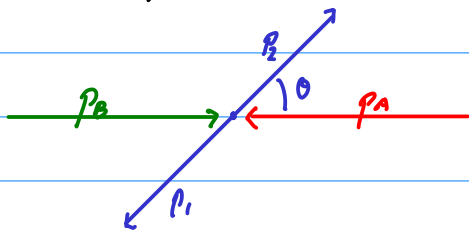
not Lorentz-inv. but inv. to boosts along the z-direction. Same transf. properties as area.

$$dT = \frac{1}{2m_A} d\pi_n |\mathcal{M}(m_A \rightarrow \{p_f\})|^2$$

$$d\pi_n = \left(\prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right) (2\pi)^4 \delta(p_A + p_B - \sum p_f)$$

"relativistically invariant n-body phase space"

Special case: Two particles in final state ($n=2$), use center-of-mass frame. Final momenta p_1 and p_2 .



$$\begin{aligned} \int d\pi_2 &= \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \frac{1}{2E_1 2E_2} (2\pi)^4 \delta(p_A + p_B - p_1 - p_2) \\ &= \int \frac{d^3 p_1}{(2\pi)^3} \frac{1}{2E_1} \frac{1}{2E_2} 2\pi \delta(E_{cm} - E_1 - E_2), \end{aligned}$$

$\int \frac{d^3 p_1}{(2\pi)^3} \frac{1}{2E_1} \frac{1}{2E_2} 2\pi \delta(E_{cm} - E_1 - E_2)$

$E_1 = \sqrt{\vec{p}_1^2 + m_1^2}$
 $E_2 = \sqrt{\vec{p}_1^2 + m_2^2}$

The $|\vec{p}_1|$ that makes $E_{cm} - E_1 - E_2 = 0$

$$= \int \frac{d\Omega}{(2\pi)^2} \frac{|\vec{p}_1|^2}{2E_1 2E_2} \left(\frac{2|\vec{p}_1|}{2E_1} + \frac{2|\vec{p}_1|}{2E_2} \right)^{-1} = \int d\Omega \frac{1}{16\pi^2} \frac{|\vec{p}_1|}{\frac{1}{E_1} + \frac{1}{E_2}}$$

$$= \int d\Omega \frac{1}{16\pi^2} \frac{|\vec{p}_1|}{E_{cm}} = \int \sin\theta d\theta d\phi \frac{1}{16\pi^2} \frac{|\vec{p}_1|}{E_{cm}} = \int d\theta \sin\theta \frac{1}{8\pi} \frac{|\vec{p}_1|}{E_{cm}}$$

$\uparrow$ if symmetry about collision axis

$\vec{p}_A + \vec{p}_B = \vec{p}_1 + \vec{p}_2 = 0$
 $E_A + E_B = E_1 + E_2 \equiv E_{cm}$

$E_i = \sqrt{\vec{p}_i^2 + m_i^2}, \quad \vec{p}_2 = -\vec{p}_1$
 $\delta(f(x)) = \frac{1}{|f'(x)|} \delta(x - x_0)$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{cm}} = \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{|\vec{p}_1|}{16\pi^2 E_{\text{cm}}} |\mathcal{M}(p_A p_B \rightarrow p_1 p_2)|^2$$

Assume $m_A = m_B = m_1 = m_2 = m \Rightarrow E_A = E_B = E_{\text{cm}}/2 = E_1 = E_2$, $E_1 = \sqrt{|\vec{p}_1|^2 + m^2} = E_2$
 $E_A = \sqrt{|\vec{p}_A|^2 + m^2} = E_B$

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{\text{cm}} &= \frac{|\vec{p}_A|}{64\pi^2 \frac{E_{\text{cm}}^2}{4} \cdot \frac{2|\vec{p}_A|}{E_{\text{cm}}/2} \cdot E_{\text{cm}}} |\mathcal{M}|^2 \\ &= \frac{|\mathcal{M}|^2}{64\pi^2 E_{\text{cm}}^2} \end{aligned}$$

$$v_A = -v_B \Rightarrow |v_A - v_B| = 2v_A$$

$$v_A = \frac{p_A}{E_A} \leftarrow \text{Energy-momentum: } E = \gamma m c^2, \quad \vec{p} = \gamma m \vec{v}$$