

UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in: FYS4170 — Relativistic Quantum Field Theory

Day of examination: 11. december 2024

Examination hours: 09.00 – 13.00

This problem set consists of 11 pages.

Appendices: Formula sheets are included.

Permitted aids: No examination support material is allowed.

Make sure that your copy of this examination paper is complete before answering.

Problem 1

a) For the following quantities, state if they are scalars, pseudo-scalars, four-vectors, pseudo-four-vectors, or none of these. Momentum vector $\mathbf{p}$ in 3D space, Dirac field ψ^α , $\bar{\psi}\psi$, $\bar{\psi}\gamma^5\psi$, γ^5 , γ^μ , electric field $\mathbf{E}$, magnetic field $\mathbf{B}$. Given: Under parity the Dirac field transforms as $\psi \mapsto \gamma^0\psi$ and $\bar{\psi} \mapsto \bar{\psi}\gamma^0$.

Solution: $\mathbf{p}$ none, ψ^α none, $\bar{\psi}\psi$ scalar, $\bar{\psi}\gamma^5\psi$ pseudo-scalar, γ^5 none, γ^μ none, electric field $\mathbf{E}$ none, magnetic field $\mathbf{B}$ none.

b) We want to consider an electron at rest at some point in space. Write down a state representing an electron (approximately) at rest and (partially) localized at some point in space. Discuss briefly its properties and behavior.

Solution: Since the electron is to be (partially) localized at some point, it must be a wave packet single-particle state:

$$|\xi, s\rangle = a_\xi^{s\dagger}|0\rangle, \quad a_\xi^{s\dagger} = \int \frac{d^3p}{(2\pi)^3} \xi(\mathbf{p}) a_{\mathbf{p}}^{s\dagger}, \quad (1)$$

where $a_\xi^{s\dagger}$ should be viewed as a wavepacket creation operator. The wavepacket spectrum $\xi(\mathbf{p})$ can be for example a normalized 3D gaussian. The electron is then localized to a region in space of width roughly given as the inverse width of the spectrum $\xi(\mathbf{p})$. The electron must spread slightly with time, since it has a spectrum of momenta. In that sense, it cannot be completely at rest. Thus, in principle, there is a trade-off between the statement “localized at a point” and “at rest”.

c) Do all electrons belong to the same quantum field, or do we define separate fields for each electron?

Solution: All electrons belong to the same Dirac field, that is, they are constructed by the ladder operators $a_{\mathbf{p}}^{s\dagger}$ from this field expression. Distinguishable particles, on the other hand, belong to different fields.

Problem 2

Consider a self-interacting theory with a classical complex scalar field ϕ :

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi \phi^* - \lambda(\phi \phi^*)^2. \quad (2)$$

a) Show that $\phi \mapsto e^{-i\alpha} \phi$, where α is a constant, is a symmetry, and show that the corresponding conserved charge is

$$Q = -i \int d^3x (\dot{\phi}^* \phi - \phi^* \dot{\phi}). \quad (3)$$

Solution: By substituting into (2) and using the fact that α is constant, we find that $\mathcal{L}$ is unchanged under the transformation $\phi \mapsto e^{-i\alpha} \phi$. Thus the transformation is a symmetry.

Noether's theorem gives the conserved current

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta \phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^*)} \Delta \phi^* = (\partial^\mu \phi^*)(-i\phi) + (\partial^\mu \phi)(i\phi^*). \quad (4)$$

The total charge Q is given by

$$Q = \int d^3x j^0 = -i \int d^3x (\dot{\phi}^* \phi - \phi^* \dot{\phi}). \quad (5)$$

b) After quantization, we have quantum fields ϕ and $\phi^\dagger$ rather than the classical fields ϕ and ϕ^* . We now consider the special case of no self-interaction ($\lambda = 0$):

$$\begin{aligned} \phi(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} (a_{\mathbf{p}} e^{-ip \cdot x} + b_{\mathbf{p}}^\dagger e^{ip \cdot x}), \\ \phi^\dagger(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} (b_{\mathbf{p}} e^{-ip \cdot x} + a_{\mathbf{p}}^\dagger e^{ip \cdot x}), \end{aligned} \quad (6)$$

where $p^0 = E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$. Here we have two independent ladder operators $a_{\mathbf{p}}$ and $b_{\mathbf{p}}$, each of them satisfying the usual bosonic commutator relation. Independent means that all commutators between a and b operators (possibly with one of them hermitian conjugate) vanish. A state $a_{\mathbf{p}}^\dagger |0\rangle$ is called a particle, while $b_{\mathbf{p}}^\dagger |0\rangle$ is an anti-particle.

Find an expression of Q in terms of ladder operators, and give an interpretation of the result. To simplify the calculations, recall that it suffices to evaluate Q at some time $t = 0$.

Solution: After quantization, Q becomes an operator

$$Q = -i \int d^3x (\dot{\phi}^\dagger \phi - \phi^\dagger \dot{\phi}). \quad (7)$$

Substituting (6), then setting $t = 0$ and letting $\mathbf{p} \rightarrow -\mathbf{p}$ for the last exponentials, we get

$$\begin{aligned} Q &= -i \int d^3x (\dot{\phi}^\dagger \phi - \phi^\dagger \dot{\phi}) = -i \int d^3x \dot{\phi}^\dagger \phi + \text{h.c.} \\ &= -i \int d^3x \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \frac{iE_{\mathbf{p}}}{\sqrt{2E_{\mathbf{p}}}\sqrt{2E_{\mathbf{q}}}} (-b_{\mathbf{p}} + a_{-\mathbf{p}}^\dagger)(a_{\mathbf{q}} + b_{-\mathbf{q}}^\dagger) e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{x}} + \text{h.c.}, \end{aligned} \quad (8)$$

where h.c. stands for the hermitian conjugate of the preceding. Performing the integral over all space, we obtain a delta function that collapses the $\mathbf{q}$ integral, leading to

$$\begin{aligned} Q &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} [(-b_{\mathbf{p}} + a_{-\mathbf{p}}^\dagger)(a_{-\mathbf{p}} + b_{\mathbf{p}}^\dagger) + \text{h.c.}] \\ &= \int \frac{d^3p}{(2\pi)^3} (a_{\mathbf{p}}^\dagger a_{\mathbf{p}} - b_{\mathbf{p}} b_{\mathbf{p}}^\dagger). \end{aligned} \quad (9)$$

Using the commutator relation and ignoring an unimportant (infinite) constant, we end up with

$$Q = \int \frac{d^3p}{(2\pi)^3} (a_{\mathbf{p}}^\dagger a_{\mathbf{p}} - b_{\mathbf{p}}^\dagger b_{\mathbf{p}}). \quad (10)$$

The operators $a_{\mathbf{p}}^\dagger a_{\mathbf{p}}$ and $b_{\mathbf{p}}^\dagger b_{\mathbf{p}}$ are the number operators, counting the number of particles and anti-particles, respectively. This means that the expression counts the number of particles and subtracts the number of anti-particles. Hence, particles created by the $a^\dagger$ and $b^\dagger$ operators have opposite charge.

c) Determine the Feynman rules of the self-interacting theory (2), and define the propagator that naturally arises as a suitable contraction. You don't have to find an expression for the propagator. You may ignore symmetry factors and combinatorics, and you don't have to mention the universal Feynman rules (momentum conservation at vertices, and integration over undetermined momenta). For external lines, take for granted that you get factors 1, just as for the usual ϕ^4 theory. Recall the formula

$$\langle \mathbf{p}' \mathbf{k}' | S | \mathbf{p} \mathbf{k} \rangle = {}_0 \langle \mathbf{p}' \mathbf{k}' | T e^{-i \int dt H_I(t)} | \mathbf{p} \mathbf{k} \rangle_{0 \text{ connected, amputated}}, \quad (11)$$

where "0" stands for free eigenstates, $\mathbf{p}$, $\mathbf{k}$, $\mathbf{p}'$, and $\mathbf{k}'$ are the momenta of external states, and $H_I(t)$ is the interaction part of the Hamiltonian written in the interaction picture.

Solution: We have

$$H_I(t) = \lambda \int d^3x (\phi \phi^\dagger)^2, \quad (12)$$

where ϕ and $\phi^\dagger$ are the fields in the interaction picture. Expanding the exponential, and consider e.g. the second order:

$$\langle \mathbf{p}'\mathbf{k}' | S | \mathbf{p}\mathbf{k} \rangle \propto (-i\lambda)^2 \langle \mathbf{p}'\mathbf{k}' | T \int d^4x \phi \phi^\dagger \phi \phi^\dagger \int d^4y \phi \phi^\dagger \phi \phi^\dagger | \mathbf{p}\mathbf{k} \rangle_0, \quad (13)$$

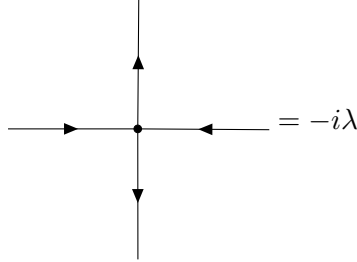
where still “connected, amputated” is implied. With the help of Wick’s theorem we get contractions, which are commutators between positive-frequency and negative-frequency parts, or alternatively, vacuum expectation values of time-ordered products of fields. From the expression for the fields ϕ and $\phi^\dagger$, we find that contractions of ϕ with itself vanish. We need therefore only

$$\overline{\phi(x)\phi^\dagger(y)} = \overline{\phi^\dagger(y)\phi(x)} = D_F(x-y), \quad (14)$$

which we can think of as the definition of a propagator $D_F(x-y)$. We are not asked to find an expression for the propagator, but it turns out to be the same as the usual propagator for hermitian, scalar fields.

For all external lines, one gets factors 1. The analysis is exactly as for the hermitian-field case. (The result is also consistent with Dirac theory, replacing $u, \bar{u}, v, \bar{v}$ by 1.)

For the vertex, we have two fields ϕ and two fields $\phi^\dagger$ in the integrals in (13). Thus each vertex will consist of four lines. Similar to the fermionic case, we have



d) Consider the process with a single vertex, with a boson–anti-boson pair as input, and also as output. (All particles have the same mass.) Determine $i\mathcal{M}$ and calculate the cross section in the center-of-mass frame. (If you did not obtain the needed Feynman rules above, just keep $\mathcal{M}$ in your expressions.)

Solution: From the Feynman vertex rule we have $i\mathcal{M} = -i\lambda$. Using the formula for the differential cross section, and integrating with the help of the two-particle phase space, the cross section is

$$\begin{aligned} \sigma &= \frac{1}{2E_A 2E_B |v_A - v_B|} \int d\Omega_{\text{cm}} |\lambda|^2 \frac{1}{16\pi^2} \frac{|\mathbf{p}_1|}{E_{\text{cm}}} \\ &= \frac{|\lambda|^2}{16\pi E_{\text{cm}}^2} \end{aligned} \quad (15)$$

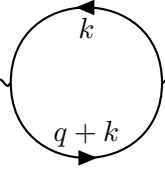
Here we have used $E_{\text{cm}} = 2E_A = 2E_B$ and $|v_A - v_B| = 2v_A = 2|\mathbf{p}_A|/E_A = 4|\mathbf{p}_1|/E_{\text{cm}}$.

Problem 3

- a) Prove that the trace of an even number of gamma matrices, $\text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \dots)$, is a sum of products of the metric.

Solution: Let n be the (even) number of gamma matrices. In $\text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \dots)$, anti-commute γ^μ to the right. For each time one picks up a term with the metric times the trace of $n - 2$ gamma matrices. Once γ^μ has reached the far end, it is moved to the front using the cyclic property of the trace. The result is then $-\text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \dots)$. One therefore finds that $\text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \dots)$ is expressed with terms of $n - 2$ gamma matrices times the metric. Continuing the process for each of these terms, one finally ends up with a sum of products of the metric.

- b) Use Feynman rules to find an expression for the vacuum polarization diagram



$$i\Pi_2^{\mu\nu}(q) = \mu \text{---} \text{wavy line } q \text{---} \text{circle} \text{---} \text{wavy line } q \text{---} \nu \quad (16)$$

Don't include propagators or external factors for the photon lines; they are just there to visualize that there are two vertices μ and ν .

Is there a problem with your expression?

Solution:

$$i\Pi_2^{\mu\nu}(q) = (-1)(-ie)^2 \int \frac{d^4k}{(2\pi)^4} \text{tr} \left[\gamma^\mu \frac{i}{\not{k} - m} \gamma^\nu \frac{i}{\not{q} + \not{k} - m} \right]. \quad (17)$$

The problem is that the integral diverges. This follows from the fact that the denominator is quadratic (k^2) for large k^2 , while the volume element is cubic.

- c) As a generalization of $i\Pi_2^{\mu\nu}(q)$ we let $i\Pi^{\mu\nu}(q)$ denote the sum of all one-particle irreducible (1PI) diagrams. Draw one example of a 1PI diagram (different from (16)) and one example of a diagram that is not 1PI.

- d) Given that $\Pi^{\mu\nu}(q)$ is a tensor, argue that it has a certain simple form. Then using that expression, argue that it can be written

$$\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu) \Pi(q^2), \quad (18)$$

for some function Π .

Solution: Since $\Pi^{\mu\nu}(q)$ is a second rank tensor, it must consist of the second rank tensors we can construct from q^μ and $g^{\mu\nu}$, which are the only tensors available in the problem. In particular, the trace of gamma matrices leads to metric terms. Thus we can write

$$\Pi^{\mu\nu}(q) = C q^\mu q^\nu + D g^{\mu\nu}. \quad (19)$$

write (25) as

$$\frac{e_0^2(-ig_{\mu\nu})}{q^2(1-\Pi(0))} = \frac{e^2(-ig_{\mu\nu})}{q^2}, \quad (26)$$

where we have defined a “physical charge”

$$e = \frac{e_0}{\sqrt{1-\Pi(0)}}. \quad (27)$$

Thus we obtain an expression that is independent of regularization by sacrificing e_0 : After all e_0 is just the “bare charge” from the Lagrangian; it is not physically measurable. Thus we can let it be dependent on regularization such that it cancel the regularization dependence of $\Pi(0)$. The result is a physical charge e and an expression (26) that are independent of the regularization parameter. This expression turns out to be e^2 times the propagator.

Formulas

Lagrangian field theory:

$$\begin{aligned}
 S &= \int d^4x \mathcal{L}(\phi, \partial_\mu \phi) \\
 \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} &= 0 \\
 \pi &= \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \\
 H &= \int d^3x (\pi \dot{\phi} - \mathcal{L})
 \end{aligned}$$

Noether:

$$\begin{aligned}
 \phi_r &\mapsto \phi_r + \alpha \Delta \phi_r \\
 \mathcal{L} &\mapsto \mathcal{L} + \alpha \partial_\mu J^\mu \\
 j^\mu &= \sum_r \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_r)} \Delta \phi_r - J^\mu
 \end{aligned}$$

Lorentz transformations, boost along the z -axis:

$$\begin{aligned}
 t' &= \gamma(t + \beta z) \\
 z' &= \gamma(z + \beta t)
 \end{aligned}$$

Klein-Gordon field:

$$\begin{aligned}
 \phi(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \left(a_{\mathbf{p}} e^{-ipx} + a_{\mathbf{p}}^\dagger e^{ipx} \right) \\
 (\partial^2 + m^2)\phi &= 0 \\
 [a_{\mathbf{p}}, a_{\mathbf{q}}^\dagger] &= (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q}).
 \end{aligned}$$

Dirac field:

$$\begin{aligned}
 \psi(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \sum_r \left(a_{\mathbf{p}}^r u^r(\mathbf{p}) e^{-ipx} + b_{\mathbf{p}}^{r\dagger} v^r(\mathbf{p}) e^{ipx} \right) \\
 (i\not{\partial} - m)\psi &= 0 \\
 \{a_{\mathbf{p}}^r, a_{\mathbf{q}}^{s\dagger}\} &= \{b_{\mathbf{p}}^r, b_{\mathbf{q}}^{s\dagger}\} = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q}) \delta^{rs} \\
 u^s(p) &= \begin{bmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \bar{\sigma}} \xi^s \end{bmatrix}, \quad v^s(p) = \begin{bmatrix} \sqrt{p \cdot \sigma} \xi^s \\ -\sqrt{p \cdot \bar{\sigma}} \xi^s \end{bmatrix} \\
 \bar{u}^r(p) u^s(p) &= -\bar{v}^r(p) v^s(p) = 2m \delta^{rs} \\
 \not{p} u(p) &= m u(p), \quad \bar{u}(p) \not{p} = \bar{u}(p) m, \\
 \not{p} v(p) &= -m v(p), \quad \bar{v}(p) \not{p} = -\bar{v}(p) m \\
 \sum_{s=1}^2 u^s(p) \bar{u}^s(p) &= \not{p} + m, \quad \sum_{s=1}^2 v^s(p) \bar{v}^s(p) = \not{p} - m
 \end{aligned}$$

Chiral representation:

$$\begin{aligned}
 \gamma^0 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}; \\
 \sigma^1 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}
 \end{aligned}$$

Definitions:

$$\begin{aligned}\overleftrightarrow{f\partial_0 g} &= f\partial_0 g - (\partial_0 f)g \\ \sigma^\mu &\equiv (1, \sigma^i), \quad \bar{\sigma}^\mu \equiv (1, -\sigma^i) \\ \not{a} &\equiv \gamma^\mu a_\mu \text{ (4-vector } a^\mu)\end{aligned}$$

Identities:

$$\begin{aligned}\sigma^i \sigma^j &= \delta^{ij} + i\epsilon^{ijk} \sigma^k \\ \{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu} \\ \gamma^5 &= i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = -\frac{i}{4!} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \\ \gamma^\mu \gamma_\mu &= 4 \\ \gamma^\mu \gamma^\nu \gamma_\mu &= -2\gamma^\nu \\ \gamma^\mu \gamma^\nu \gamma^\alpha \gamma_\mu &= 4g^{\nu\alpha} \\ \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\rho \gamma_\mu &= -2\gamma^\rho \gamma^\alpha \gamma^\nu \\ \text{tr}[\gamma^\mu \gamma^\nu] &= 4g^{\mu\nu} \\ \text{tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] &= 4g^{\mu\nu} g^{\rho\sigma} - 4g^{\mu\rho} g^{\nu\sigma} + 4g^{\mu\sigma} g^{\nu\rho} \\ \text{tr}[\gamma^{\mu_1} \dots \gamma^{\mu_n}] &= 0, \text{ for } n \text{ odd}\end{aligned}$$

Ward's identity:

Let $i\mathcal{M}_\mu(q)$ be a diagram where a photon line with four-momentum q at vertex μ is removed. Then $\mathcal{M}_\mu(q)q^\mu = 0$.

Gordon's identity:

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p') \left[\frac{p'^\mu + p^\mu}{2m} + \frac{i\sigma^{\mu\nu} q_\nu}{2m} \right] u(p), \text{ where } \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \text{ and } q^\mu = p'^\mu - p^\mu.$$

Cross-sections and decay rates:

$$\begin{aligned}d\sigma &= \frac{1}{2E_A 2E_B |v_A - v_B|} \left(\prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right) \\ &\quad \times |\mathcal{M}(p_A, p_B \rightarrow \{p_f\})|^2 (2\pi)^4 \delta^4(p_A + p_B - \sum p_f) \\ d\Gamma &= \frac{1}{2m_A} \left(\prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right) \\ &\quad \times |\mathcal{M}(p_A = (m_A, \mathbf{0}) \rightarrow \{p_f\})|^2 (2\pi)^4 \delta^4(p_A - \sum p_f)\end{aligned}$$

Two-particle phase space:

$$\int \frac{d^3 p_1}{(2\pi)^3} \frac{1}{2E_1} \frac{d^3 p_2}{(2\pi)^3} \frac{1}{2E_2} (2\pi)^4 \delta^4(p_A + p_B - p_1 - p_2) = \int d\Omega_{\text{cm}} \frac{1}{16\pi^2} \frac{|\mathbf{p}_1|}{E_{\text{cm}}},$$

$E_{\text{cm}} = E_1 + E_2$ in the center-of-mass frame; Ω_{cm} = solid angle for particle 1. For decay of a single particle, set $p_B = 0$.

Lagrangians and Feynman rules

The following rules are used to obtain the amplitude $i\mathcal{M}$, and given in momentum space. Dots represent vertices in all of the diagrams listed below. For consistency between ϕ^4 theory and Yukawa theory, scalar particles are represented by dashed lines, and their masses by M , in both cases.

Common for all theories:

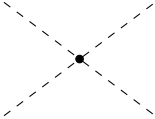
- Impose momentum conservation at each vertex
- Integrate over each undetermined momentum, $\int \frac{d^4k}{(2\pi)^4}$

 ϕ^4 theory

$$\mathcal{L}_{\phi^4} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}M^2 \phi^2 - \frac{\lambda}{4!} \phi^4$$

• Propagator: $\bullet \xrightarrow{p} \bullet = \frac{i}{p^2 - M^2 + i\epsilon}$

• External line: $\xrightarrow{p} \bullet = 1$

• Vertex:  $= -i\lambda$

- Divide by the symmetry factor

Yukawa theory:

$$\mathcal{L}_Y = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}M^2 \phi^2 + \bar{\psi}(i\not{\partial} - m)\psi - g\bar{\psi}\psi\phi$$

• Scalar propagator: $\bullet \xrightarrow{p} \bullet = \frac{i}{p^2 - M^2 + i\epsilon}$

• Fermion propagator: $\bullet \xrightarrow{p} \bullet = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$

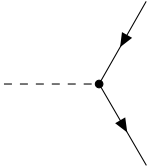
• External scalar: $\xrightarrow{p} \bullet = 1$

• Incoming fermion (spin s): $\xrightarrow{p} \bullet = u^s(p)$

• Incoming antifermion (spin s): $\xleftarrow{p} \bullet = \bar{v}^s(p)$

• Outgoing fermion (spin s): $\bullet \xrightarrow{p} = \bar{u}^s(p)$

• Outgoing antifermion (spin s): $\bullet \xleftarrow{p} = v^s(p)$

• Vertex:  $= -ig$

- Work out the overall sign of the diagram
- For closed fermion loops, include a factor (-1) and a trace.

QED:

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\not{\partial} - e\not{A} - m)\psi - \frac{1}{4}(F_{\mu\nu})^2,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad e = -|e|$$

- Photon propagator: $\mu \xrightarrow{p} \nu = \frac{-ig_{\mu\nu}}{p^2 + i\epsilon}$
- Fermion propagator: $\bullet \xrightarrow{p} \bullet = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$
- Incoming photon: $\mu \xrightarrow{p} = \epsilon_\mu(p)$
- Outgoing photon: $\bullet \xrightarrow{p} \mu = \epsilon_\mu^*(p)$
- Incoming fermion (spin s): $\xrightarrow{p} \bullet = u^s(p)$
- Incoming antifermion (spin s): $\xleftarrow{p} \bullet = \bar{v}^s(p)$
- Outgoing fermion (spin s): $\bullet \xrightarrow{p} = \bar{u}^s(p)$
- Outgoing antifermion (spin s): $\bullet \xleftarrow{p} = v^s(p)$
- Vertex: $\mu \xrightarrow{\quad} \bullet = -ie\gamma^\mu$
- Work out the overall sign of the diagram
- For closed fermion loops, include a factor (-1) and a trace.