

Opamp supplementary

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Abstract

A very brief supplementary to the official syllabus-defining literature. This supplementary is describing operational amplifiers very briefly along with some nice circuits.

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1 Introduction

1.1 Why this supplementary?

A demand for a brief supplementary to other sources of knowledge about operational amplifiers (opamps) has been voiced, and this is an attempt to make such a supplementary. It is only introductory, and is by any mean not aiming to be a reference document.

There are literally tons of books that cover opamps better, more thoroughly, more elegant, and so on; so go ahead and study the subject more if you like. If you only want one book on electronics, and one that covers a practical approach, you should consider *The Art of Electronics*, third edition, by Paul Horowitz and Winfield Hill (<https://artofelectronics.net>). If you just want to look at some cool opamp applications, try this:

https://en.wikipedia.org/wiki/Operational_amplifier_applications.

1.2 Why opamps?

Most physiological quantities we want to have a look at start as a physical quantity - utterly useless and inaccessible for the physician. But don't despair - there are ways of transforming the physical quantities to something that can be displayed on a screen. The way it typically happens is like this:

1. Physical quantity.
2. Transducer convert physical quantity to analogue electrical quantity => Current, voltage, charge, etc.
3. Processing of electric quantity:
 - a. Transformation between electrical representations (current, voltage, charge, etc.).
 - b. Amplification.
 - c. Summation.
 - d. Filtering.
 - e. Other mathematical stuff.
4. Conversion form analogue representation to digital representation.
5. Processing by computer.
6. Display of result.

It is in the processing of the analogue signals opamps play an important role since opamps with a few external connections and components can perform most required mathematical operations on the signals.

2 *Laws and very basic introductory stuff*

In much the same way as we have to obey laws, electrons have to obey laws too. The most important ones are presented here.

2.1 *Ohm's law*

$$U = RI \quad (1)$$

Any resistor will behave according to this law. The positive side is, by definition, where the current enters.

The impedance (Z_R) of a resistor is always the same as it's resistance, R .

2.2 *Kirchhoff's first law*

The sum of all currents into a node is zero.

This simply means that charge is conserved, and that an electric node cannot store charge in any way.

2.3 *Murphy's law*

What can go wrong, will.

Just for fun. But there is a touch of experience here saying that it is important to keep circuits as small and simple as possible.

2.4 *The capacitor*

If you know anything about capacitors, please don't read this since it is a veeeeery simplified view of how we can use a capacitor.

OK, so you don't know much about capacitors. That's fine. Here is the information you need to enable you to read this text.

Some useful formulas when looking at the capacitor in the time domain:

$$U = \frac{1}{C} \int I \, dt \quad (2)$$

$$I = C \frac{dU}{dt} \quad (3)$$

Equations 2 and 3 are really the same equations after some fiddling. There should of course be a constant in equation 2, but it's left out for simplicity. The equations show that the relation between U and I is somewhat different from the relation for resistors as described by Ohm's law. As for resistors, the positive side of the voltage across the capacitor is on the side the current enters.

If we consider a capacitor in the frequency domain, it has some funny properties. It turns out that the capacitor can be seen as a device with frequency dependent impedance. Impedance is an expansion of the resistance concept, and an impedance value is actually a vector in the imaginary plane. We are only going to consider the vector length, or impedance magnitude here. The frequency dependence of the capacitor impedance is given by

$$|Z_C| = \frac{1}{2\pi fC}. \quad (4)$$

The essence of equation 4 is that the capacitor is having high impedance at low frequencies and low impedance at high frequencies. This is summarized in table 1 along with the impedance of the resistor.

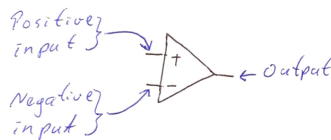
Device	DC (f=0 Hz)	Low frequency	High frequency	Infinite frequency
$ Z_R $	R	R	R	R
$ Z_C $	∞	High	Low	0

Table 1: Impedance for resistors and capacitor for some frequencies.

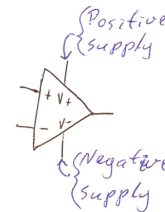
3 The opamp

3.1 The ideal opamp

The opamp is a device made of a number of other components like transistors, resistors, and capacitors neatly packaged into a small package. For our convenience, suppliers of opamps make them in a variety of sizes and with a variety of capabilities. The basic opamp is a three-pin device as shown in figure 1a), while in reality it has at least five pins as shown in figure 1 b). We will use the three-pin symbol for simplicity.



a) Opamp, three pin symbol.



b) Opamp, five pin symbol.

Figure 1: Opamp symbols.

The basic function is described by equation 5

$$U_{OUT} = A_D(U_{IN+} - U_{IN-}) \quad (5)$$

where U_{OUT} is the output voltage, U_{IN+} is the voltage at U_{IN+} , U_{IN-} is the voltage at the U_{IN-} , and A_D is the differential gain. A_D is typically very large, so large that it is pointless to use the opamp without any form of feedback. A_D can easily be in the range 1000 to 1000000, and is commonly expressed in decibels ¹.

3.2 CMRR and PSRR

Since opamps are not ideal, we have to look at some more features. From equation 5, we can be led to believe that opamps are ignoring common mode voltages. Well, they usually do a pretty good job at it, but they are not perfect. A common mode voltage is a voltage that is present on both inputs. As a result, we have to expand equation 6 a bit, and get:

¹ Decibel is a logarithmic scale, and for signal values (not power values), it is defined as $A_{dB} = 20\log_{10}(A)$.

$$U_{OUT} = A_D(U_{IN+} - U_{IN-}) + A_{CM} \left(\frac{U_{IN+} + U_{IN-}}{2} \right) \quad (6)$$

where A_{CM} is the common mode gain. A_{CM} is usually very low, and can easily be less than 0.01. Since we are interested in knowing how much the common mode voltage is rejected, we are usually talking about the *Common Mode Rejection Rate* (CMRR) which usually is expressed in decibels, and given by

$$CMRR = \frac{A_D}{|A_{CM}|} = 20 \log_{10} \left(\frac{A_D}{|A_{CM}|} \right) \text{ dB}. \quad (7)$$

A signal on the power supplies² may also be transferred to the output. Once again, we have to expand the equation describing the opamp, and get

$$U_{OUT} = A_D(U_{IN+} - U_{IN-}) + A_{CM} \left(\frac{U_{IN+} + U_{IN-}}{2} \right) + A_{PS} U_{PS} \quad (8)$$

where A_{PS} is the gain of a signal on the power supply and U_{PS} is the unwanted signal that lies on top of the power supply voltage. The ratio of rejection is of interest, and similarly to CMRR, Power Supply Rejection Rate (PSRR) is given by

$$PSRR = \frac{A_D}{|A_{PS}|} = 20 \log_{10} \left(\frac{A_D}{|A_{PS}|} \right) \text{ dB}. \quad (9)$$

3.3 Some other properties

Some properties of the opamp are presented in the table below.

Property	Ideal opamp	Real-world opamp
Input impedance	Infinite.	Very high.
Output impedance (output is seen as a voltage source)	Zero.	From 1Ω to 100Ω.
Differential gain	Infinite.	Very high.
Common mode gain	Zero.	Very low.
Power supply gain	Zero.	Very low.
Bandwidth	Infinite.	Finite.
Noise	Zero.	Yes.

Table 2: Opamp properties for ideal and real-world opamps.

4 Some cool opamp circuits

4.1 Analysis

Analysis is left to the readers³. To simplify analysis of a sensible designed opamp circuit, there are two rules of thumb that will make it easy:

- The voltages on both inputs are identical if some sort of feedback is used.
- There will not flow any current into the inputs.

These rules of thumb apply for all circuits here.

The two most common way to describe how a circuit is processing a signal are an output function or by a transfer function. An output function simply states what the output is, and a

² The signal is typically noise or interference added to the pure and somewhat ideal DC voltage on the power supply.

³ Lecturers and authors just love this sentence. First of all, it's fun, and then it's a cue to the serious reader that this is smart if she/he/it want to learn the stuff.

transfer function must be multiplied by an input signal. For example, equation 10 is an output function, and equation 11 is an transfer function of the same circuit.

$$U_{OUT} = U_{IN} \left(1 + \frac{R_1}{R_2} \right) \quad (10)$$

$$H = \frac{U_{OUT}}{U_{IN}} = 1 + \frac{R_1}{R_2} \quad (11)$$

4.2 Non-inverting amplifier

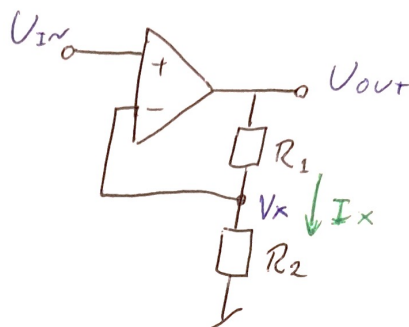


Figure 2: Non-inverting amplifier.

Parameter	Value / Description
Transfer function	$\frac{U_{OUT}}{U_{IN}} = 1 + \frac{R_1}{R_2}$
Input impedance	Very high.
Output impedance	Very low.
Pros	Not inverting.
Cons	Gain under 1 impossible.

Table 3: Opamp circuit properties for non-inverting amplifier.

4.3 Unity gain buffer

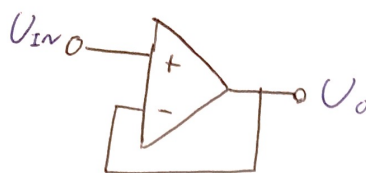


Figure 3: Unity-gain buffer.

Parameter	Value / Description
Transfer function	$\frac{U_{OUT}}{U_{IN}} = 1 + \frac{0}{\infty} = 1$
Input impedance	Very high.
Output impedance	Very low.
Pros	Buffer.
Cons	May add noise.

Table 4: Opamp circuit properties for unity-gain buffer amplifier.

4.4 Inverting amplifier

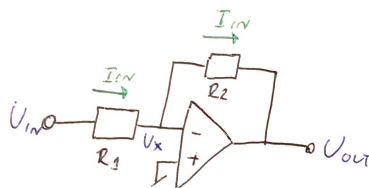


Figure 4: Inverting amplifier.

Parameter	Value / Description
Transfer function	$\frac{U_{OUT}}{U_{IN}} = -\frac{R_2}{R_1}$
Input impedance	$=R_1$
Output impedance	Very low.
Pros	Large range of gain possible, even below 1. Low output impedance.
Cons	Inverting. Low input impedance.

Table 5: Opamp circuit properties for inverting amplifier.

4.5 Integrator / low-pass filter

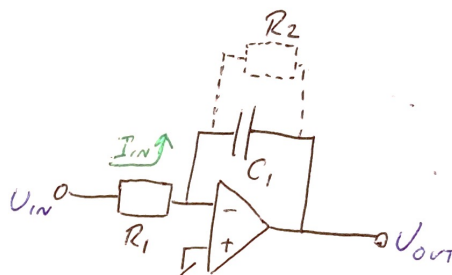


Figure 5: Integrator / low pass filter.

Parameter	Value / Description
Output function	$U_{OUT} = -\frac{1}{C_1} \int I_{IN} dt = -\frac{1}{C_1} \int \frac{U_{IN}}{R_1} dt = -\frac{1}{R_1 C_1} \int U_{IN} dt$
Input impedance	$=R_1$
Output impedance	Very low.
Pros	Large range of gain possible. Low output impedance.
Cons	Inverting. Low input impedance. Need R_2 or some other form of discharging of C_1 to prevent build-up of charge.

Table 6: Opamp circuit properties for an integrator / low-pass filter.

As an exercise, look at the transfer function for the inverting amplifier, and think of the capacitor as an impedance with values that varies according to table 1. Which frequencies are let through, and which are stopped?

4.6 Derivator / high-pass filter

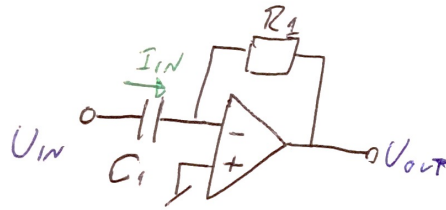


Figure 6: Derivator / high-pass filter.

Parameter	Value / Description
Output function	$U_{OUT} = -R_1 I_{IN} = -R_1 C_1 \frac{dU}{dt}$
Input impedance	Capacitive.
Output impedance	Very low.
Pros	Large range of gain possible. Low output impedance.
Cons	Inverting. Capacitive/low input impedance.

Table 7: Opamp circuit properties for a derivator / high-pass filter.

As an exercise, look at the transfer function for the inverting amplifier, and think of the capacitor as an impedance with values that varies according to table 1. Which frequencies are let through, and which are stopped?

4.7 Logarithmic and exponential amplifier

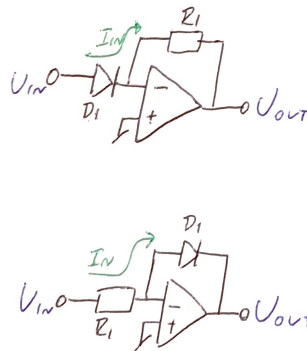


Figure 7: Exponential and logarithmic amplifiers.

Parameter	Value / Description
Output function	$U_{OUT} = -R_1 K \exp(U_{IN})$ $U_{OUT} = -K \ln\left(\frac{U_{IN}}{R_1}\right)$
Input impedance	Low $= R_1$
Output impedance	Very low.
Pros	Cool circuits. Low output impedance. Non-linear effects may be useful.
Cons	Inverting. Inaccurate and temperature dependent. Low input impedance.

Table 8: Opamp circuit properties for two related non-linear circuits.

Since it may be that a reader does not know everything there is to know about diodes, equation 12 describing the voltage-current relation of a diode:

$$I_D = I_S \left(e^{\frac{U_D}{\eta V_T}} - 1 \right). \quad (12)$$

If we simplify, and accept that the constants are just offsets and annoying numbers, we can reduce equation 12 to something more useful, as shown in equations 13 and equation 14.

$$I_D = K e^{U_D} \quad (13)$$

$$U_D = K \ln(I_D) \quad (14)$$

This means that we can implement the functions $\ln X$ and e^X . And since we can do addition (see summing amplifier below), we can implement multiplication since $e^{\ln A + \ln B} = AB$. How cool is that?

4.8 Summing amplifier

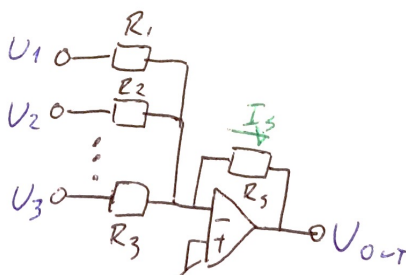


Figure 8: Summing amplifier.

Parameter	Value / Description
Output function	$U_{OUT} = -R_S \sum \frac{U_n}{R_n}$
Input impedance	$= R_n$
Output impedance	Very low.
Pros	Large range of gain possible. Low output impedance.
Cons	Inverting. Low input impedance.

Table 9: Opamp circuit properties for summing amplifier. It is possible to adjust gain of each input by changing the value of the corresponding input resistor.

4.9 Differential amplifier

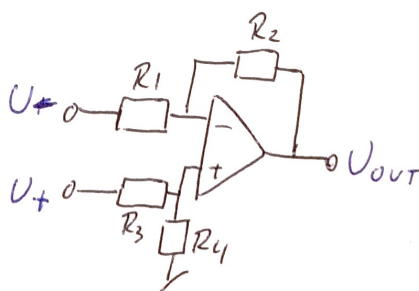


Figure 9: Differential amplifier.

Parameter	Value / Description
Output function	$U_{OUT} = \frac{R_{2/4}}{R_{1/3}}(U_+ - U_-)$
Input impedance	$= R_{1/3} + R_{2/4}$
Output impedance	Very low.
Pros	Low output impedance. Accurate if gain is low.
Cons	Low input impedance. Large error if gain is high. Poor CMRR, especially if gain is high.

Table 10: Opamp circuit properties for differential amplifier.

4.10 Instrumentation amplifier

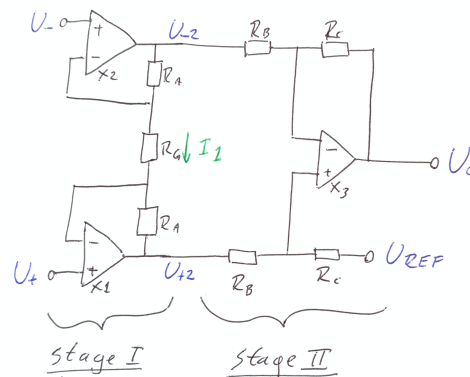


Figure 10: An instrumentation amplifier.

This is a beauty, so it will be explained in a bit more detail.

The circuit has two stages, and the input stage is providing differential gain and high input impedance. Our rule of thumb says that the voltages on the opamp inputs are identical, which means that U_+ and U_- are the voltages on each side of R_G . I_1 can now be calculated

$$I_1 = \frac{U_- - U_+}{R_G} \quad (15)$$

Since no currents are flowing into the - inputs of the two opamps, we know that I_1 is passing through the resistor chain, and we can use Ohm's law to calculate the voltage across the resistor chain

$$U_{-2} - U_{+2} = I_1(2R_A + R_G) = \frac{U_- - U_+}{R_G}(2R_A + R_G) \quad (16)$$

Since the voltage across the resistor chain is the same as the output of stage I, we can do some fiddling on equation 16, and we see that the differential gain of stage I is given by

$$A_{D(\text{Stage I})} = 1 + \frac{2R_A}{R_G} \quad (17)$$

Since the common mode voltage on the output of stage I is the same as the input, the stage I common mode gain is 1.

Now that we have a nicely amplified input signal available after stage I, we want to get rid of the common mode voltage. This is done in stage II by using a differential amplifier where gain is set to 1 (or close to 1 in order to reduce noise and error and such). Note that there is a pin called

U_{REF} . This is typically placed at some reference level like ground, but it can be any reference level you choose. The point is that the output voltage U_{OUT} is relative to this.

The output function is then

$$U_{OUT} = (U_+ - U_-) \left(1 + \frac{2R_A}{R_G}\right) \quad (18)$$

relative to the potential on U_{REF} .

Both stages combined will then:

- have very low input impedance.
- have a differential gain which can be set by changing R_G .
- have a very low common mode gain.
- be a differential amplifier on steroids.

Lucky for us, there are a load of these commercially available.

4.11 Active filter examples (Sallen & Key)

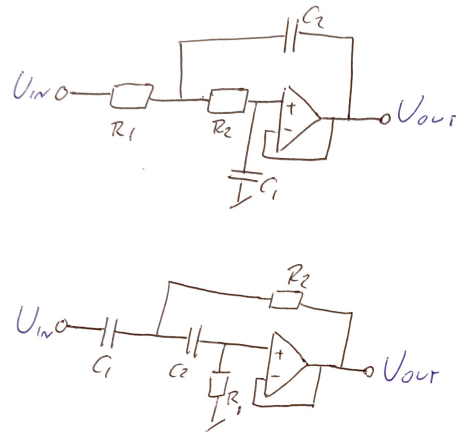


Figure 11: Sallen and Key filters. The top circuit is a low-pass filter and the bottom circuit is a high-pass filter.

As an example of filters using opamps, the Sallen & Key filters are nice. These are second order filters with an opamp amplifier. They can be both low-pass and high-pass. It is very common to use the same value on both resistors and both capacitors. More information here:

https://en.wikipedia.org/wiki/Sallen-Key_topology.

5 A more accurate analysis of the inverting opamp circuit

The rules of thumb can be a bit frustrating for some of you, so here is a more thorough analysis for one circuit just to show how it can be done. Similar analyses can be done for all circuits.

The equations that describe the circuit are

$$U_X = U_{OUT} + I_{in}R_2 = U_{OUT} + \frac{U_{IN} - U_{OUT}}{R_1 + R_2} R_2 \quad (19)$$

and

$$U_{OUT} = -U_X A_D. \quad (20)$$

By inserting 20 into 19, we get

$$-\frac{U_{OUT}}{A_D} = U_{OUT} + U_{IN} \frac{R_2}{R_1+R_2} - U_{OUT} \frac{R_2}{R_1+R_2} \quad (21)$$

which can be manipulated into

$$\frac{U_{OUT}}{U_{IN}} = -\frac{A_D R_2}{A_D R_1 + R_1 + R_2}. \quad (22)$$

If we assume that A_D is very large, say ∞ , this reduces to the familiar form

$$\frac{U_{OUT}}{U_{IN}} = -\frac{R_2}{R_1}. \quad (23)$$

If we assume that A_D is very large, and that the output is within reasonable limits, equation 5 tell us that the inputs must be very close to each other, and if A_D approaches ∞ , the potential difference between U_{IN+} and U_{IN-} approaches 0.