

High energy physics Experiments - Part II

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FYS4505

[A. Read](#)

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Important concepts

- Radiation length
- Interaction length
- Electromagnetic showers
- Hadronic showers
- Multiple scattering (of charged particles)
- Tracking and vertex detectors
- Momentum resolution of magnetic spectrometer
- Energy resolution of calorimeters (EM, hadronic)
- Particle identification (TOF, Cherenkov, transition, muon ID)

High Energy particle interactions

- Electrons above ~ 10 MeV lose energy mainly by radiation

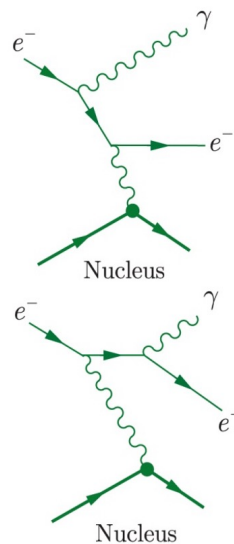


Figure 4.11 Dominant Feynman diagrams for the bremsstrahlung process
 $e^- + (Z, A) \rightarrow e^- + \gamma + (Z, A)$.

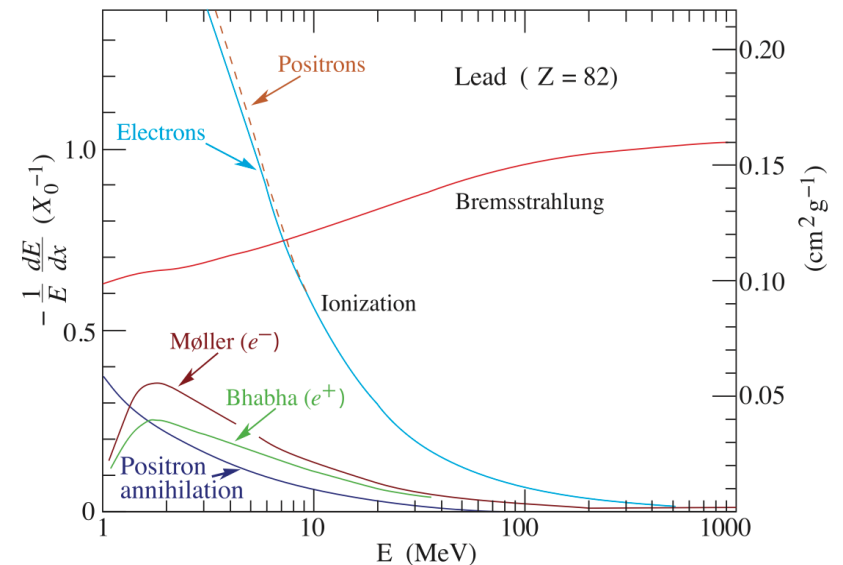


Figure 34.11: Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Møller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, *Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers*, Pergamon Press, 1970. Messel and Crawford use $X_0(\text{Pb}) = 5.82 \text{ g/cm}^2$, but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials ($X_0(\text{Pb}) = 6.37 \text{ g/cm}^2$).

Radiation length

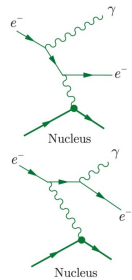


Figure 4.11 Dominant Feynman diagrams for the bremsstrahlung process $e^- + (Z, A) \rightarrow e^- + \gamma + (Z, A)$.

Nuclear and Particle Physics: An Introduction, Third Edition, B.H. Martin and G. Shaw,
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- For electrons with $E \gg m_e c^2 / Z^{1/3}$
- $-dE/dx = E/X_0 \Rightarrow E = E_0 e^{-x/X_0}$
- Radiation length is distance over which the energy of a high-energy electron is reduced by $1/e$ (on average).
- A_r is atomic weight relative to $1/12$ of ^{12}C
- For a detector to be transparent/opaque, what sort of materials do we choose?

$$\frac{1}{X_0} \approx 4\alpha r_0^2 \frac{\rho N_A}{A_r} Z_{\text{nucl}}(1 + Z_{\text{nucl}}) \ln \left(\frac{183}{\sqrt[3]{Z_{\text{nucl}}}} \right)$$

Table 2.1 Radiation length X_0 for some common materials

Material	Radiation length X_0
Air	304 m
Water	36 cm
Shielding concrete	10.7 cm
Nylon	36.7 cm
Aluminium (Al)	8.9 cm
Silicon (Si)	9.36 cm
Iron (Fe)	1.76 cm
Lead (Pb)	0.56 cm
Uranium (U)	0.32 cm

Radiation length and γ 's

- For high-energy photons ($E \gg m_e c^2 / Z^{1/3}$) the pair-production cross-section, which is dominant, can be approximated as

$$\sigma(\gamma Z_{nucl} \rightarrow e^+ e^- Z_{nucl}) \approx \frac{7}{9} \frac{1}{n_a X_0}$$

- What is n_a ?
- Photon attenuation $dI = -I \frac{dx}{\lambda}$, $\lambda = \frac{1}{n_a \sigma_\gamma}$
- $I(x) = I_0 e^{-x/\lambda} \Rightarrow I(x) = e^{-7x/(9X_0)}$
- Photon absorption and electron energy loss both characterized by $\sim$ radiation length X_0

Electromagnetic showers

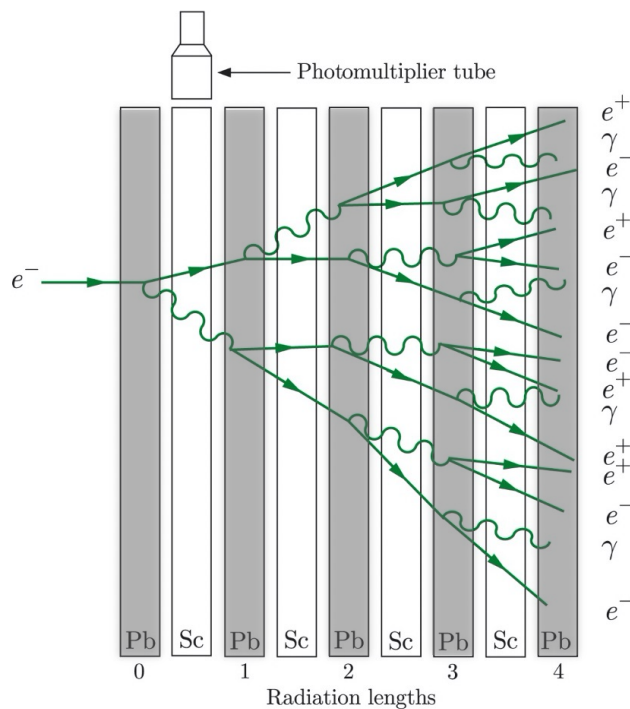
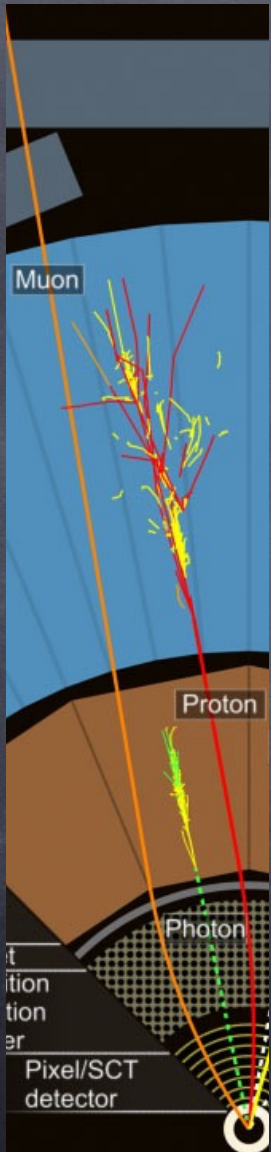


Figure 4.20 Approximate development of an electromagnetic shower in a sampling calorimeter assuming the simple model of the text. The calorimeter consists of alternate layers of lead (Pb) and a scintillator (Sc), the latter attached to photomultipliers (one only shown).

- Assume energy is shared 50-50 at each interaction
- After t X_0 layers the photons and electrons will have energy $E(t) = E_0/2^t$
- Allow process to repeat until critical energy $E_c \approx 600 \text{ MeV}/Z$
- $t_{max} = t(E_c) = \frac{\ln(E_0/E_c)}{\ln 2}$
- $N_{max} = E_0/E_c$
- How does the thickness of a calorimeter scale with the maximum energy to detect?

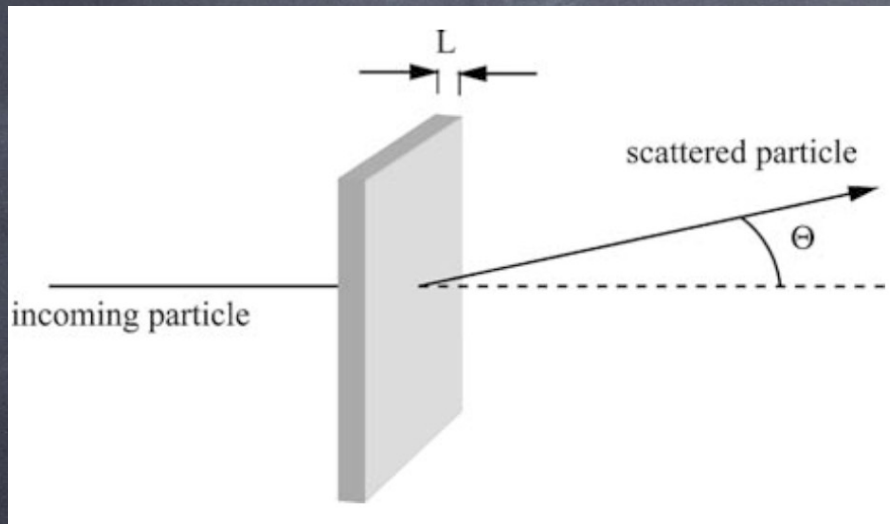
Interaction length

- Probability of (hadronic) interaction $= n\sigma_{tot}dx$
- Mean distance between interactions $l_c = 1/(n\sigma_{tot})$
- Typically $l_c \gg X_0$ (recall $X_0(Fe) \approx 2$ cm)
- Interaction lengths $\sim 40-10$ cm for 100-300 GeV nucleons interacting with metals between aluminium and lead
- $\pi^0 \rightarrow \gamma\gamma$ in the hadronic shower cause EM fluctuations
- Energy resolution of HCAL tends to be much worse than ECAL
- «Compensating calorimeters» use uranium as absorber -
can you reason why?



Multiple scattering

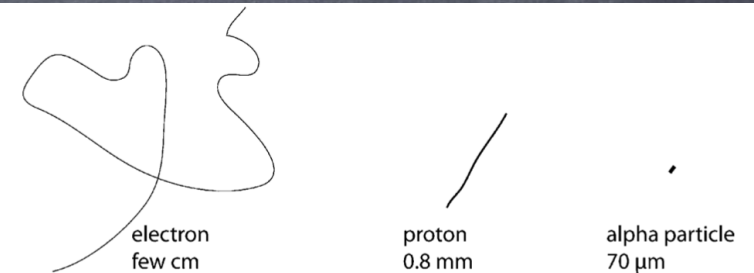
$$\sqrt{\langle \Theta^2 \rangle} = \frac{Z}{Pc\beta} (20 \text{ MeV}) \sqrt{\frac{L}{X_0}}$$



Θ = scattering angle relative to the incoming particle in radians
 P = momentum of the incoming particle
 X_0 = radiation length of the material

- Discuss the trajectories...

Fig. 2.9 A typical trajectory for an electron, a proton and an alpha particle of 10 MeV in silicon. The electron trajectory is drawn on a scale 10 times smaller than the trajectory of the proton and the alpha particle



Momentum measurement

- Lorentz force on charged particle in magnetic field (SI units):

$$\vec{F} = \dot{\vec{p}} = q\vec{v} \times \vec{B}$$

- Relativistic kinematics: $\vec{p} = m\gamma\vec{\beta}c = \frac{m\gamma c^2}{c^2}\vec{v} = \frac{E\vec{v}}{c^2}$

- Since $\vec{F} \perp \vec{v}$, no work is done, therefore, $E = \text{constant}$: $\dot{\vec{p}} = \frac{E}{c^2}\dot{\vec{v}}$

For constant $\vec{B}$ there is a constant perpendicular force, the motion will be circular, and $v = \text{constant}$: Let $\vec{v} = v(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$ and $\vec{B} = \hat{z}B$

- $\dot{\vec{p}} = \frac{E}{c^2}\dot{\vec{v}} = \frac{E}{c^2}v\omega(-\hat{x}\sin\omega t + \hat{y}\cos\omega t) = qvB(\hat{x}\sin\omega t - \hat{y}\cos\omega t)$

- Up to a sign we have $\omega = \frac{qc^2B}{E}$

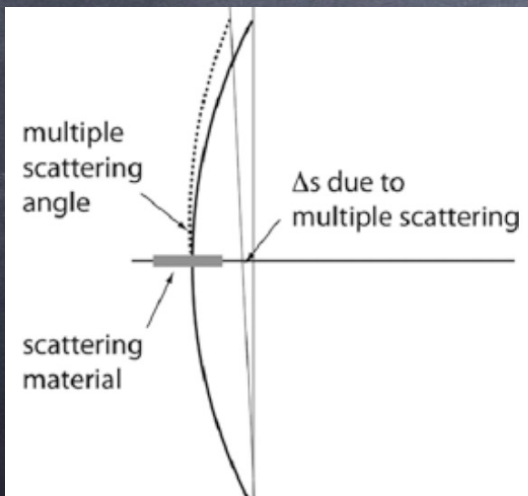
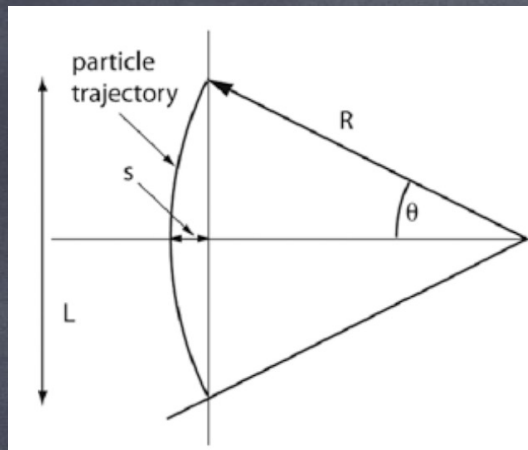
Momentum measurement

- From previous page: $\omega = \frac{qc^2B}{E}$
- In the plane $\perp$ to $\vec{B}$ we have $\vec{p} = \frac{E\vec{v}}{c^2} \rightarrow p_{\perp} = \frac{Ev_{\perp}}{c^2} = \frac{ER\omega}{c^2} = qRB$

Momentum measurement

- $p_{\perp} = qRB$
- Suppose we want momentum in GeV (natural units), R in m, B in Tesla (T), and $q = Ze$
- $cp_{\perp} = Z \cdot 1.6022 \times 10^{-19} \text{ C} \cdot 3 \times 10^8 \text{ m/s} \frac{R(\text{m})B(\text{T})}{1.6022 \times 10^{-19} \text{ J/eV}}$
- $p_{\perp} (\text{GeV}) \simeq 0.3 Z R(\text{m})B(\text{T})$

Momentum measurement



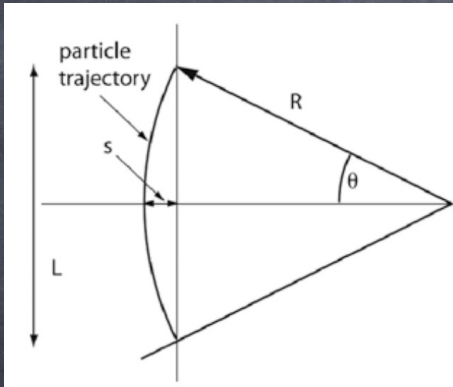
- Typically follow charged particles with tracking detectors (drift chambers, TPC, silicon strips and pixels) in a uniform magnetic field
- ALICE, ATLAS, CMS all have central solenoid with $\vec{B} \parallel$ beams.
- The tracks follow helixes (circles in the plane $\perp$ to $\vec{B}$) with $pc(\text{GeV}) = 0.3 Z B(T)R(m)$, i.e. $p \propto R$
- Sagitta $s = (1 - \cos\theta)R \approx \theta^2 R/2 \approx L^2/(8R)$

Momentum resolution

We have

$$s \approx L^2 / (8R)$$

and $pc = 0.3ZBR$



We therefore have

$$\frac{\sigma\{s\}}{s} = \frac{\sigma\{R\}}{R} = \frac{\sigma\{P\}}{P}$$

$$\frac{\sigma\{P\}}{P} = 8\sigma\{s\} \frac{Pc}{0.3 Z B L^2}$$

In the case of only three equidistant points along the trajectory, the error on the sagitta is related to the error on the position measurement of the points σ by

$$\sigma\{s\} = \sqrt{\sigma^2 + \frac{\sigma^2}{2}} = \sqrt{\frac{3}{2}}\sigma$$

The momentum resolution due to the measurement error on the space points is therefore given by

$$\left[\frac{\sigma\{P\}}{P} \right]_{SP} = 8\sqrt{\frac{3}{2}} \frac{\{Pc\}[\text{GeV}]}{0.3 Z B[\text{tesla}] L^2[\text{m}]} \sigma$$

It can be shown that, if there are N equidistant points along the trajectory, this expression generalises to (see [6])

$$\left[\frac{\sigma\{P\}}{P} \right]_{SP} = \sqrt{\frac{720(N-1)^3}{(N-2)N(N+1)(N+2)}} \frac{\{Pc\}[\text{GeV}]}{0.3 Z B[\text{tesla}] L^2[\text{m}]} \sigma$$

- Which factors improve/worsen the momentum resolution?
- Why don't experiments necessarily use the maximum B allowed by cost?
- What is large- N dependence and how does this influence choice of N ?

Stop 22.11..23

Multiple scattering (again)

The contribution of the multiple scattering to the momentum resolution is hence given by

$$\left[\frac{\sigma\{P\}}{P} \right]_{MS} = \sqrt{\frac{2}{3}} \frac{0.02}{0.3} \frac{1}{L[\text{m}]B[\text{tesla}] \beta} \sqrt{\frac{L}{X_0}}$$

Assumes 3 measurements and uniform distribution of material.

- What should we consider when designing a detector?
- Compare the dependence on momentum to the term from measurement uncertainty.

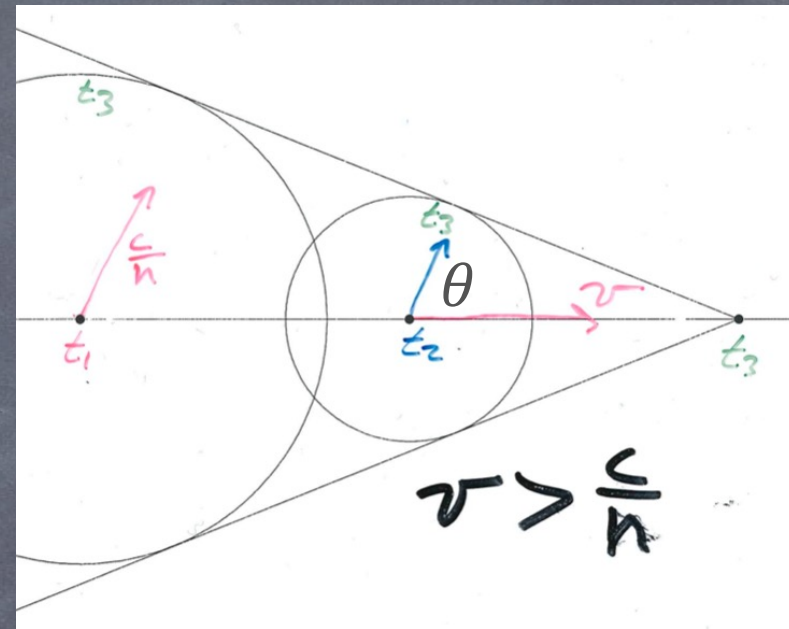
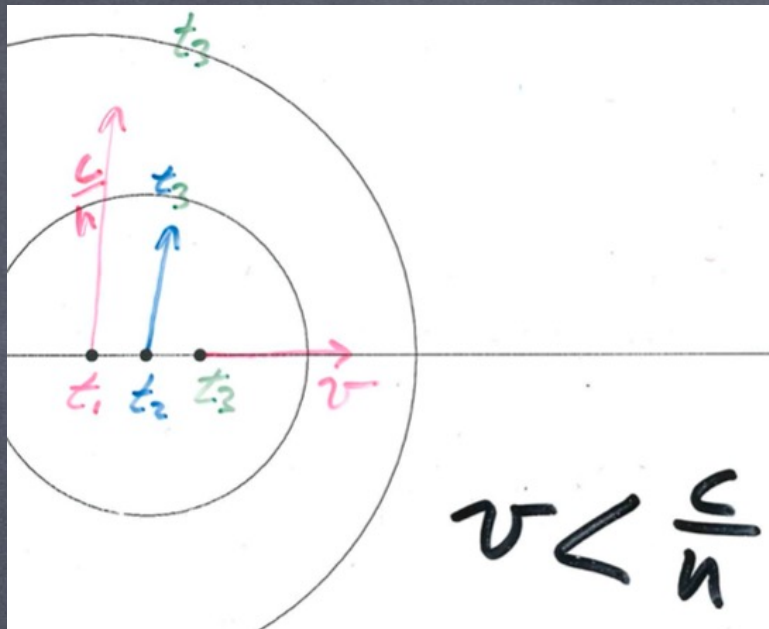
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Calorimeter energy resolution («lite»)

- Typically of the form $\left(\frac{\sigma_E}{E}\right)^2 = \left(\frac{a}{\sqrt{E}}\right)^2 + b^2$
- For sampling ECAL $a=10-30\%$
- For sampling HCAL (only reasonable solution) $a=60-100\%$
- For homogenous ECAL (various crystals, lead-glass) $a=1-5\%$
- A good calorimeter has $b<1\%$
 - CMS lead-tungstate crystals has $a=3\%$, $b=0.5\%$

Cherenkhov detectors



- Čerenkhov radiation analogous to shock wave when objects move faster than the speed of sound in a gas
- Huygen's construction: $\cos\theta = \frac{\frac{c}{n}t}{\beta ct} = \frac{1}{n\beta}$
- Gases - liquids - glasses: $1 < n < 2$

Can we get Č in vacuum?
So, when do we see Č radiation?

Cherenkov detectors

The intensity of the Cherenkov effect can be calculated from first principles by solving the Maxwell equations with the proper boundary conditions. The result of this calculation is

$$\frac{d^2 E}{d\hbar\omega \cdot dx} = \hbar\omega \frac{Z^2 \alpha}{\hbar c} \left[1 - \frac{c^2}{n^2 v^2} \right] \quad v > \frac{c}{n}$$
$$\frac{d^2 E}{d\hbar\omega \cdot dx} = 0 \quad v < \frac{c}{n}$$

In the above equation, the notation is as follows:

Z = charge of the particle in units 'proton charge'

E = energy emitted in the form of optical photons

n = optical refractive index

c = velocity of light in vacuum

v = velocity of the particle

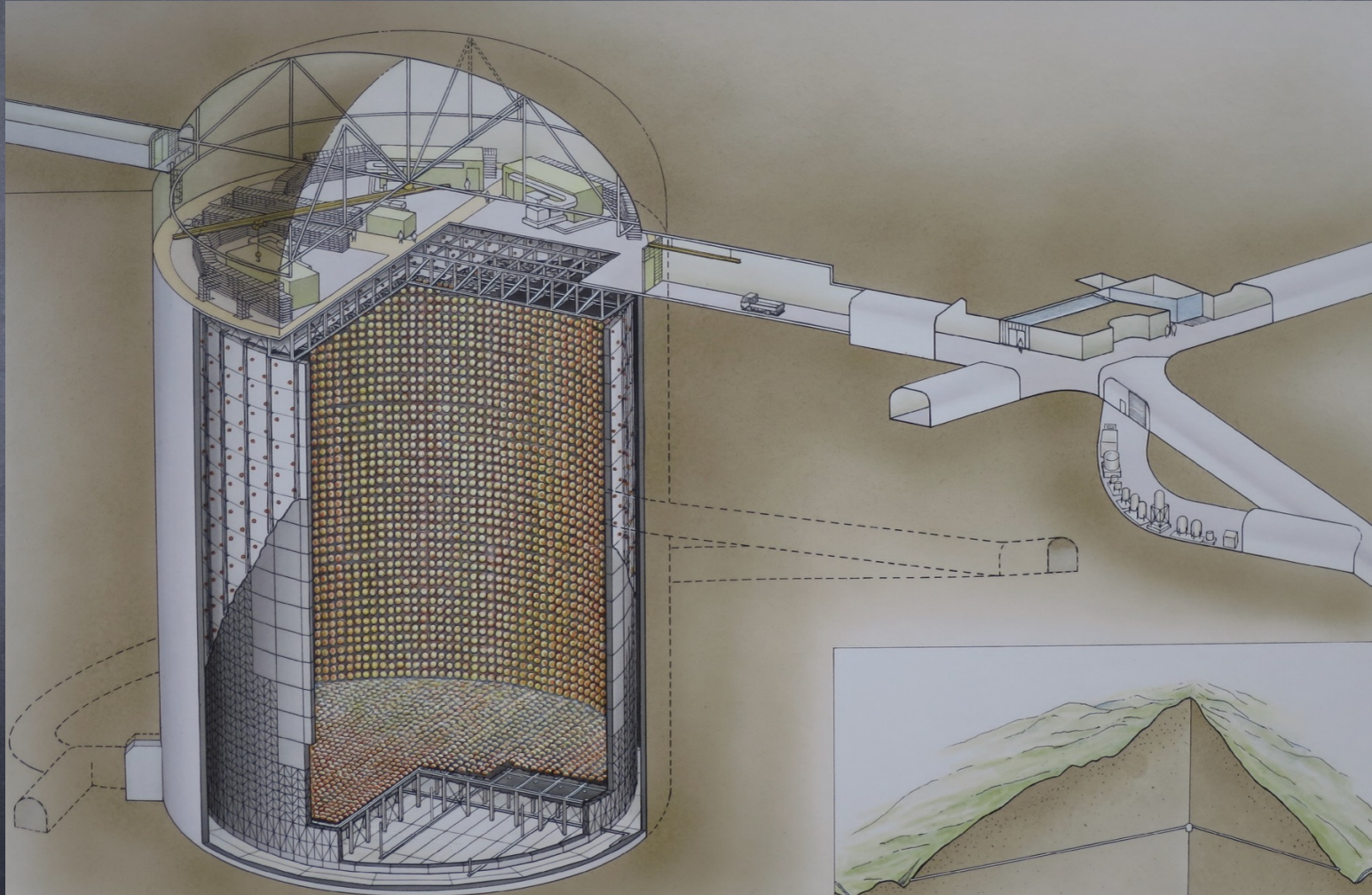
$\hbar\omega$ = energy of the emitted photon

α = fine structure constant (1/137)

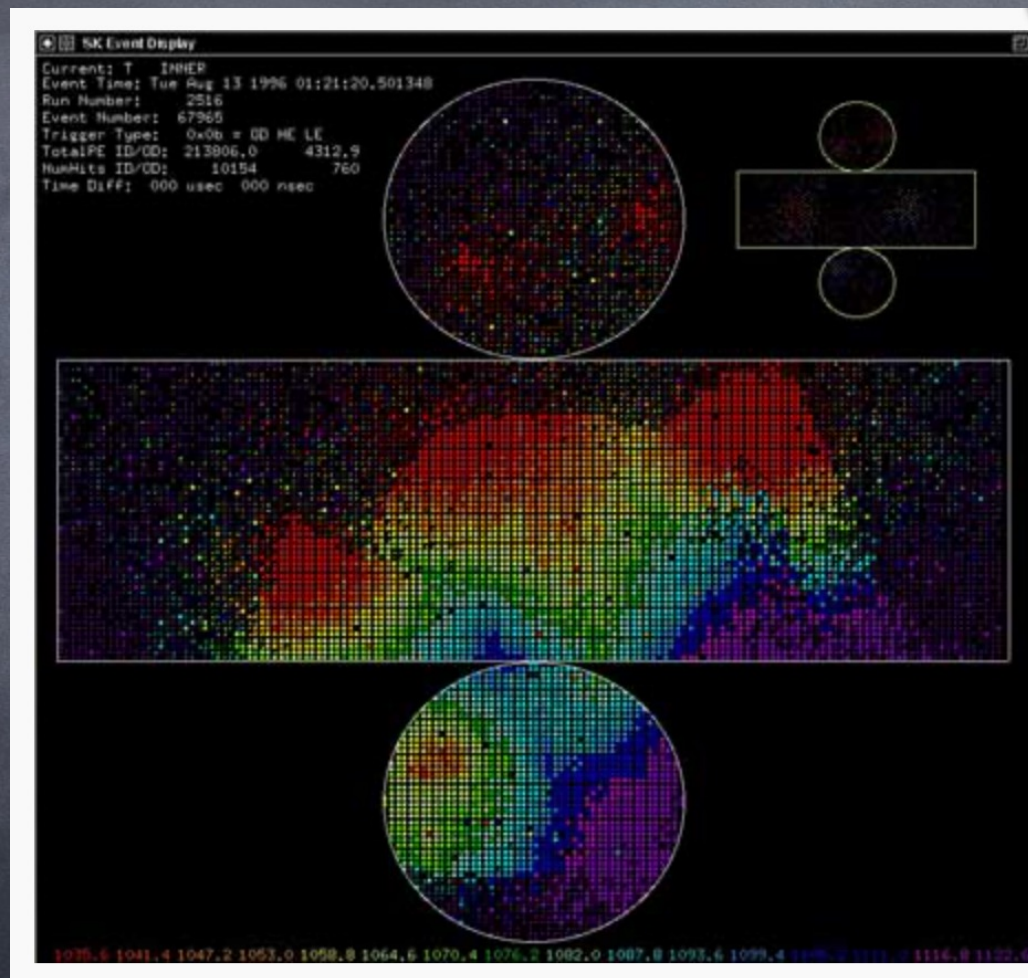
$\hbar c$ = numerical constant of value $197 \cdot 10^{-9}$ eV m

- Divide by $\hbar\omega$ to get number of photons per energy interval per distance.
- 1 GeV electron gives 220 visible photons/cm in water and 30/m in air.

Super Kamiokande



3-muon event in Super-K



RICH: Ring Imaging CHerenkov detector

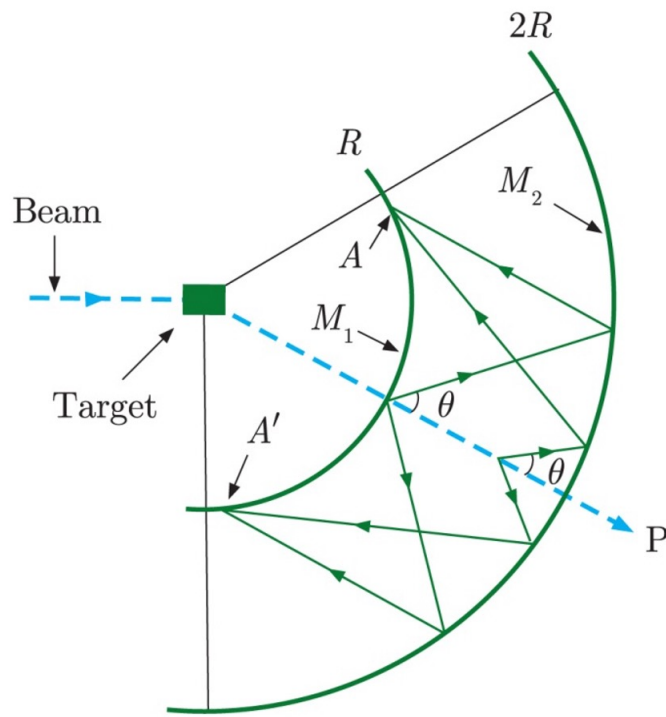


Figure 4.19 A particle P, produced from the target, emits Čerenkov radiation on traversing a medium contained between two spheres of radius R and $2R$. The mirror M_2 on the outer sphere focuses the radiation into a ring image at AA' on the inner detector sphere M_1 . The radii of the ring image depend on the angle of emission of the Čerenkov radiation and hence on the velocities of the particles.

- Threshold effect: $\beta > 1/n$
- Size of ring: $\cos\theta = \frac{1}{n\beta}$
- Number of photons: $\propto Z^2$ and $f(\beta)$

Time of flight (TOF)

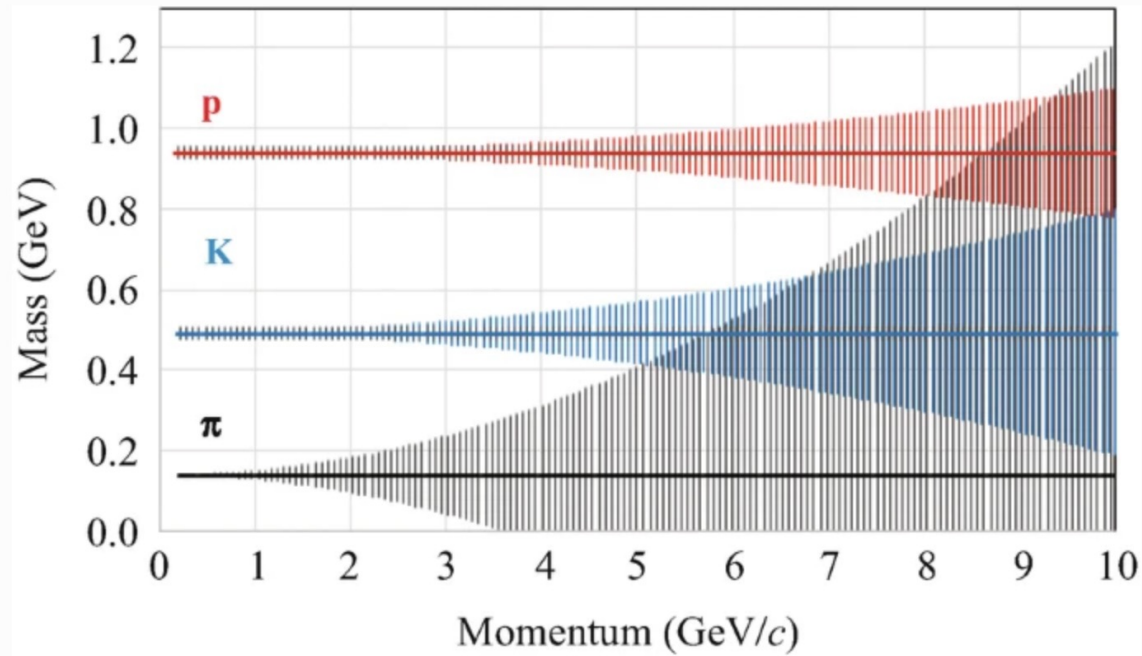
- Measure momentum in magnetic spectrometer and speed with pair of timing detectors: Allows us to measure mass, or at least support or rule out various mass hypotheses with respect to each other.
- What makes good and simple timing detectors?
- 2 planes separated by L , particles of momentum p
- $\text{TOF} \equiv t = \frac{L}{\beta c}, \beta = \frac{L}{ct}$
- $p = mc\gamma\beta, p^2 = m^2c^2\gamma^2\beta^2 = \frac{m^2c^2\beta^2}{1-\beta^2}$
- $m^2 = \frac{p^2(1-\beta^2)}{\beta^2} \frac{1}{c^2} = p^2 \left(\frac{c^2t^2}{L^2} - 1 \right) \frac{1}{c^2}$

Time of flight (TOF)

- Time resolution is often dominant uncertainty: $2m\Delta m \approx p^2 2t\Delta t/L^2$
- Substitute $t = \frac{L}{c\beta}$ and divide by $m = \frac{p}{c\gamma\beta}$: $\Delta m \approx \frac{c\gamma\beta}{p} \frac{p^2}{L^2} \frac{L}{c\beta} \Delta t$
- $\Delta m \approx \gamma p \frac{\Delta t}{L}$
- Mass resolution improves with larger L , worsens with larger p
- Mass resolution for fixed p worsens with larger γ
- $\gamma = \frac{E}{mc^2} = \frac{\sqrt{m^2 c^4 + p^2 c^2}}{mc^2} = \sqrt{1 + \frac{p^2}{m^2 c^2}}$
- Smaller mass for fixed p gives larger γ , therefore, worse resolution

TOF resolution (example)

Fig. 7.2



Mass resolution as function of momentum for a Time of Flight, ToF, detector with $\Delta p/p = 4 \cdot 10^{-3}$, $l = 10$ m, $\Delta l = 10^{-4}$ and $\Delta t = 50$ ps

Important concepts

- Radiation length
- Interaction length
- Electromagnetic showers
- Hadronic showers
- Multiple scattering (of charged particles)
- Tracking and vertex detectors (MWPC, drift, TPC, silicon)
- Momentum resolution of magnetic spectrometer
- Energy resolution of calorimeters (EM, hadronic)
- Particle identification (TOF, Cherenkov, transition, muon ID)

Examples of relevant questions

- What happens when a 2 GeV photon hits a large block of inorganic scintillator?
- How can we distinguish a several GeV electron from a several GeV charged pion in a multipurpose detector, without any explicit particle identification detector?