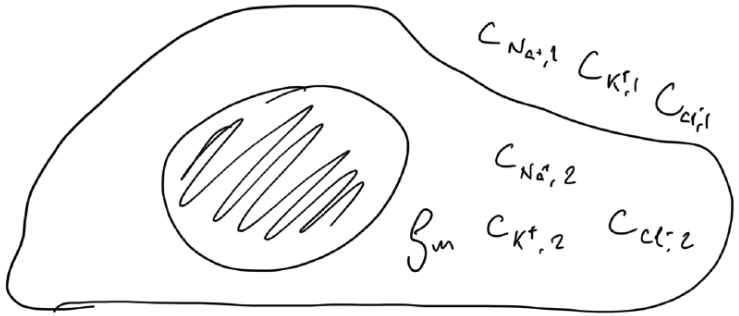
Transport through membranes

Membrane permeable to one ion species => equilibrium: $\Delta\mu = kT \ln \frac{c_1}{c_2} - q\Delta V = 0$ (Nernst)

If we assume that inside a cell we have negatively charged macromolecules (eg. proteins) that can not permeate the semipermeable cell membrane. The small ions Na^+, Cl^- and K^+ can permeate.



$\rho_{m/e} = 125mM$
$c_{K^+,1} = 10mM$
$c_{Na^+,1} = 140mM$
$c_{Cl^-,1} = 150mM$

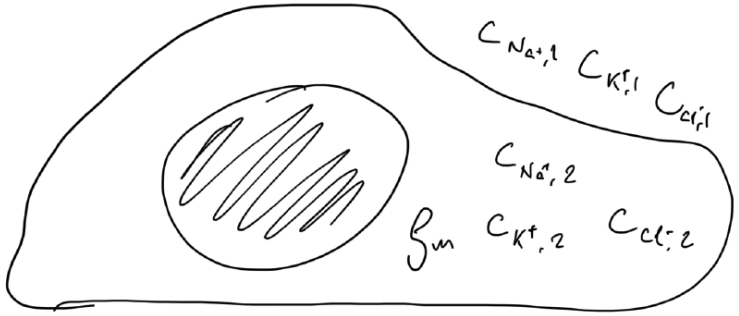
For all species that can permeate the membrane the concentration difference over the membrane must be given by the same Nernst potential:

$$\frac{e\Delta\mathcal{V}_{Nernst}}{kT} = \ln\frac{c_{Na^+,2}}{c_{Na^+,1}} = \ln\frac{c_{K^+,2}}{c_{K^+,1}} = \ln\frac{c_{Cl^-,1}}{c_{Cl^-,2}} \tag{40}$$

Exponentiating we get the so-called Donnan equilibrium:

$$\frac{c_{Na^+,2}}{c_{Na^+,1}} = \frac{c_{K^+,2}}{c_{K^+,1}} = \frac{c_{Cl^-,1}}{c_{Cl^-,2}} \tag{41}$$

Transport through membranes



$\rho_{m/e} = 125\,mM$

$c_{K^+,1} = 10\,mM$

$c_{Na^+,1} = 140\,mM$

$c_{Cl^-,1} = 150\,mM$

$$\frac{e\Delta\mathcal{V}_{Nernst}}{kT} = \ln\frac{c_{Na^+,2}}{c_{Na^+,1}} = \ln\frac{c_{K^+,2}}{c_{K^+,1}} = \ln\frac{c_{Cl^-,1}}{c_{Cl^-,2}} \tag{40}$$

Exponentiating we get the so-called Donnan equilibrium:

$$\frac{c_{Na^+,2}}{c_{Na^+,1}} = \frac{c_{K^+,2}}{c_{K^+,1}} = \frac{c_{Cl^-,1}}{c_{Cl^-,2}} \tag{41}$$

With the 2 equations of the Donnan equilibrium and the equation for charge balance inside the cell

$$0 = c_{Na^+,2} + c_{K^+,2} + c_{Cl^-,2} + \frac{\rho_m}{e} \tag{42}$$

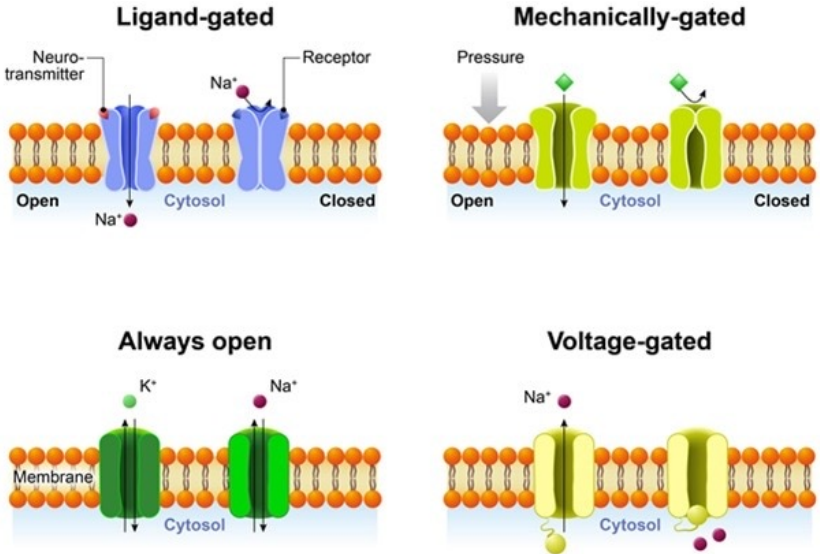
we have 3 equations and 3 unknown concentrations and can solve to find the ion concentrations inside the cell: $c_{Na^+,2} = 210$ mM, $c_{K^+,2} = 15$ mM and $c_{Cl^-,2} = 100$ mM. The proteins are few and highly charged and do not contribute to the total number of osmotic particles inside the cell $c_{tot,2} = 325$ mM. The total number of osmotic particles outside the cell is $c_{tot,1} = 300$ mM. The resulting osmotic pressure is

$$\Delta P = RT\Delta c = 8.3 \cdot 310 \cdot 25 \cdot 10^{-3} = 64 \text{ Pa} \tag{43}$$

Transport through membranes

Permeation is slow (solubility in membrane)

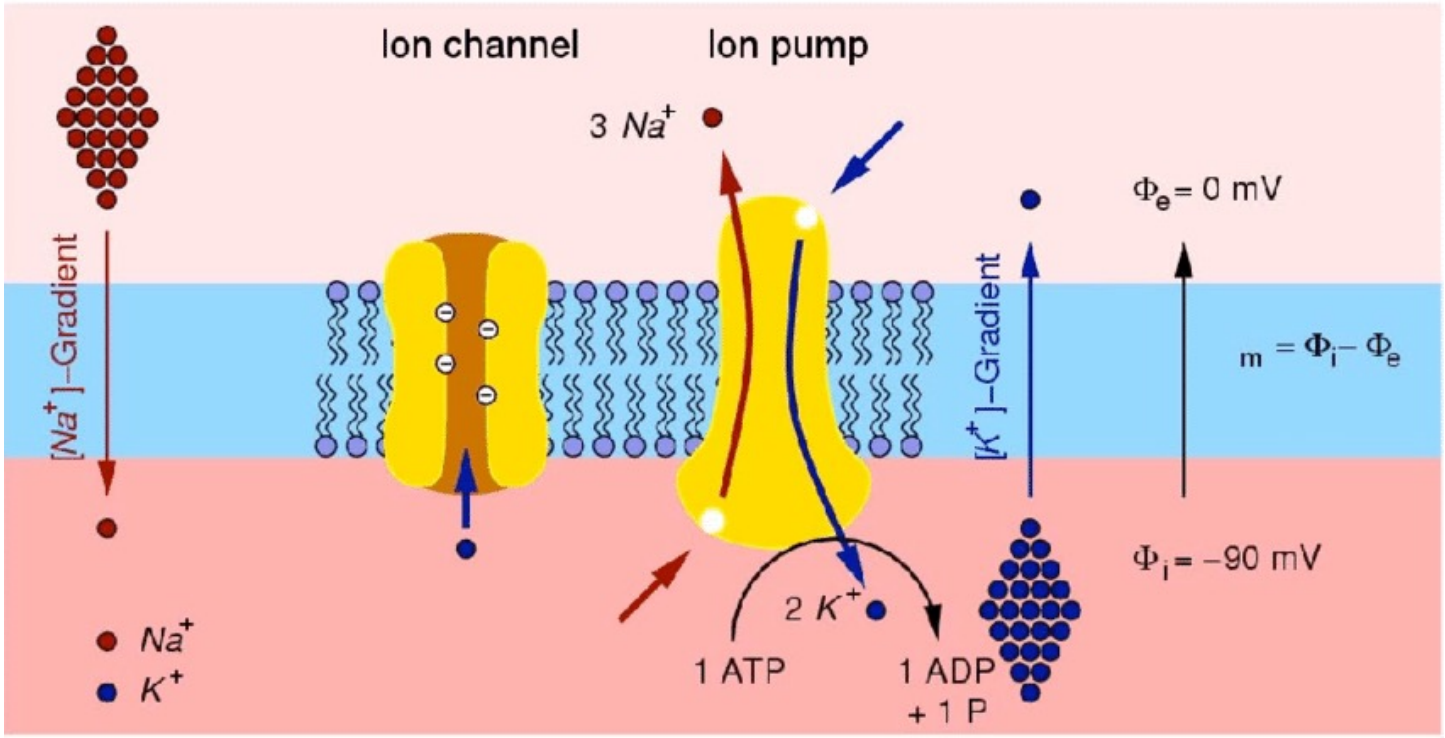
Ion channels for "passive transport": More than 106 ions are transported in a second through the ion channels without the help of metabolic energy like ATP, cotransport, or the active transport mechanism. There are *two types ion channels*, *nonselective* or large pore and *selective* (archetypal) or small pores. Ions typically pass through the channel pores in the form of a single file almost as fast as they move through a free solution. In most of the ion channels, the *passage across the pores is governed by a "gate."* The gate may be opened or closed in response to different factors such as: *electrical signals, chemical signals, temperature, and the mechanical force.* Electrical gradient across the plasma membrane are responsible for the opening and closing of voltage-gated ion channels; however, binding of the ligands to the channels is responsible for the activation and deactivation of ligand-gated ion channels (from DOI: 10.5772/intechopen.80597).



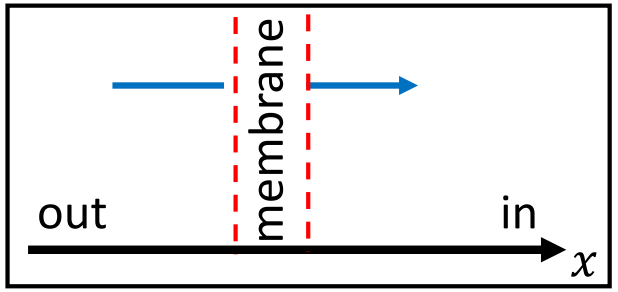
Transport through membranes

Ion pumps control *ion concentration* and *electrical potential* over cell membrane.

Pumping requires energy: ADP-> ATP



11.1-2 Ion pumping -> sodium anomaly in all animal cells



Fluxes, $j_i, j_{p,i}$ of ion species i, **out -> in** positive

Forces, $\Delta\mu_i = kT\Delta \ln c_i + q_i\Delta V + v_i\Delta P,$ **in-out** positive

Linear transport: $j_i = j_{p,i} - g_i\Delta\mu_i$

$= j_{p,i} - g_i\Delta \ln c_i + g_i q_i\Delta V + g_i v_i\Delta P$

steady state $= 0$

$j_{p,i} = \alpha_i j_p$ ion pump flux, $\alpha_{Na+} = 3, \quad \alpha_{K+} = -2, \quad j_p > 0$

j_i flux through membrane

g_i conductance through membrane

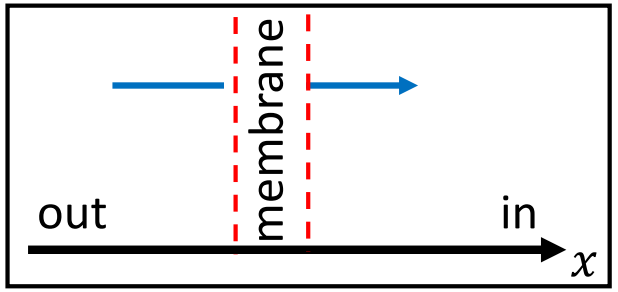
q_i charge of ion i

ΔV (electrical) membrane potential

$v_i = c_i^{-1}$ (partial) volume of ion i (ideal mixture)

$\Delta P = 0$ assume no pressure difference

11.1-2 Ion pumping -> sodium anomaly in all animal cells



Fluxes, $j_i, j_{p,i}$ of ion species i, **out -> in** positive

Forces, $\Delta\mu_i = kT\Delta \ln c_i + q_i\Delta V + v_i\Delta P$, **in-out** positive

Linear transport: $j_i = j_{p,i} - g_i\Delta\mu_i$

$= j_{p,i} - g_i\Delta \ln c_i + g_i q_i\Delta V + g_i v_i\Delta P$

steady state $= 0$

$j_{p,i} = \alpha_i j_p$ ion pump flux, $\alpha_{Na+} = 3, \quad \alpha_{K+} = -2, \quad j_p > 0$

j_i flux through membrane

g_i conductance through membrane

q_i charge of ion i

ΔV (electrical) membrane potential

$v_i = c_i^{-1}$ (partial) volume of ion i (ideal mixture)

$\Delta P = 0$ assume no pressure difference

charge neutrality inside: $c_{Na+,in} + c_{K+,in} - c_{Cl-,in} - \frac{\rho_m}{e} = 0$

flux charge neutrality: $j_{Na+} + j_{K+} + j_{Cl-} = 0$ (1)

steady state:

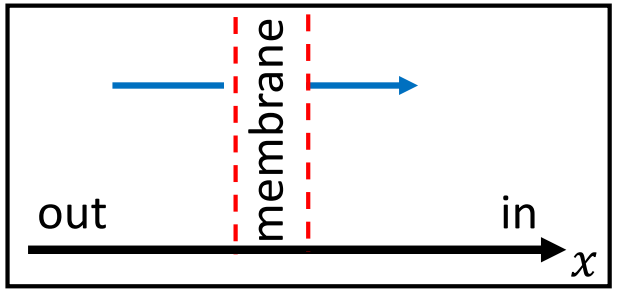
$$\frac{3j_p}{g_{Na+}} = kT \ln \frac{c_{Na+,o}}{c_{Na+,i}} + e\Delta V \quad (2)$$

$$-\frac{2j_p}{g_{K+}} = kT \ln \frac{c_{K+,o}}{c_{K+,i}} + e\Delta V \quad (3)$$

$$0 = kT \ln \frac{c_{Cl-,o}}{c_{Vl-,i}} - e\Delta V \quad (4)$$

4 equations, 4 unknowns: $c_{Na+,i}, c_{K+,i}, c_{Cl-,i}, \Delta V$

11.1-2 Ion pumping -> sodium anomaly in all animal cells



charge neutrality inside: $c_{Na+,in} + c_{K+,in} - c_{Cl-,in} - \frac{\rho_m}{e} = 0$

flux charge neutrality: $j_{Na+} + j_{K+} + j_{Cl-} = 0$ (1)

steady state: $\frac{3j_p}{g_{Na+}} = kT \ln \frac{c_{Na+,o}}{c_{Na+,in}} + e\Delta V$ (2)

$-\frac{2j_p}{g_{K+}} = kT \ln \frac{c_{K+,o}}{c_{K+,in}} + e\Delta V$ (3)

$0 = kT \ln \frac{c_{Cl-,o}}{c_{Vl-,in}} - e\Delta V$ (4)

4 equations, 4 unknowns: $c_{Na+,in}, c_{K+,in}, c_{Cl-,in}, \Delta V$

Only permeating ions not pumped $\Delta V \approx V_i^N = -\frac{kT}{q_i} \ln \frac{c_{i,o}}{c_{i,i}}$

Steady state = resting potential $\Delta V = V^0$

Book notation $g_i = g_i q_i$

11.1-2 Ion pumping -> sodium anomaly in all animal cells

$$V_i^N = -\frac{kT}{q_i}\ln\frac{c_{i,o}}{c_{i,i}}, \qquad \mathcal{g}_i = g_i e$$

flux charge neutrality: $j_{Na+} + j_{K+} + j_{Cl-} = 0$ (1)

steady state: $\frac{3j_p}{\mathcal{g}_{Na+}} = -V_{Na+}^N + \Delta V$ (2)

$$-\frac{2j_p}{\mathcal{g}_{Na+}} = -V_{K+}^N + \Delta V \quad (3)$$

$$0 = V_{Cl-}^N - \Delta V \quad (4)$$

(2)+(3) $2\mathcal{g}_{Na+}(\Delta V - V_{Na+}^N) = 3\mathcal{g}_{K+}(\Delta V - V_{K+}^N)$

$$\Delta V = \frac{2\mathcal{g}_{Na+}V_{Na+}^N + 3\mathcal{g}_{K+}V_{K+}^N}{2\mathcal{g}_{Na+} + 3\mathcal{g}_{K+}}$$

Pumping =>

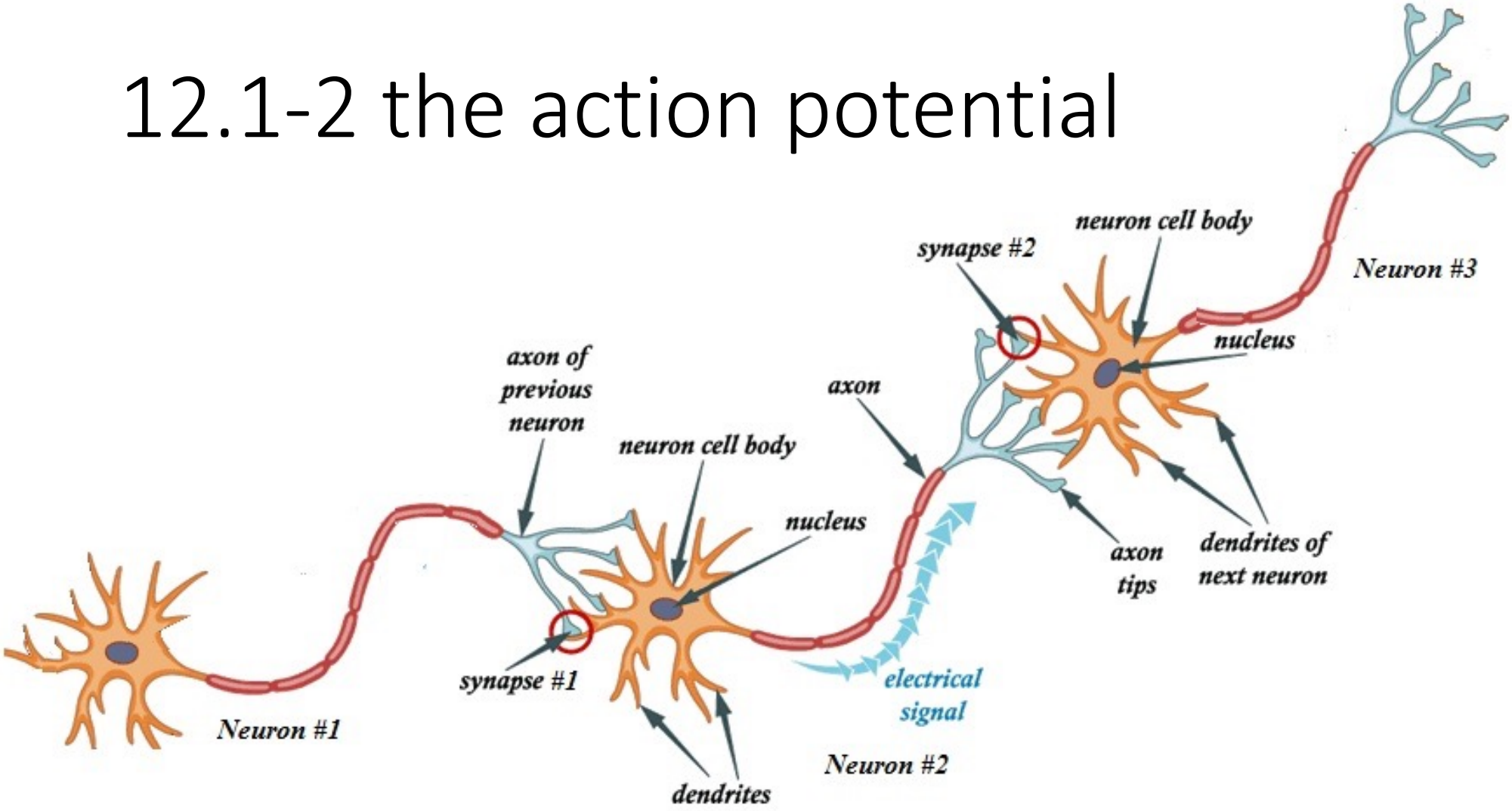
ion imbalance

	$c_{i,o}$ [mM]	$c_{i,in}$ [mM]	V_i^N [mV]	g_i/g_{K+}
K^+	20	400	-75	1
Na^+	440	50	54	$\frac{1}{25}$ (* 8)
Cl^-	560	52	-59	$^1/_2$

=> $\Delta V = -72$ mV, but eq. (4): $\Delta V = -59$ mV

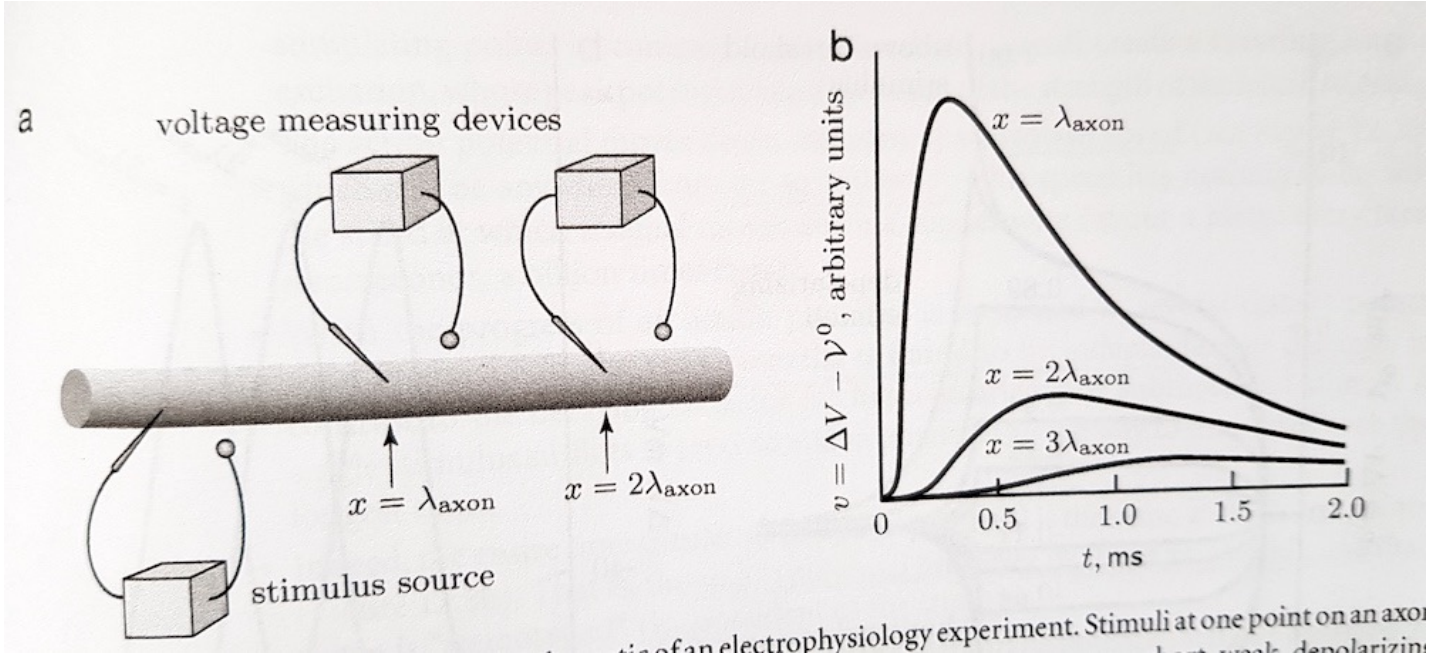
(1) can be used to correct (4), but (1) is not complete because other ions present ++

12.1-2 the action potential



- How can a leaky cable carry a sharp signal over long distances?
- Nonlinearity in cell membrane's conductance => excitable medium => regenerates signal

Nerve impulses???

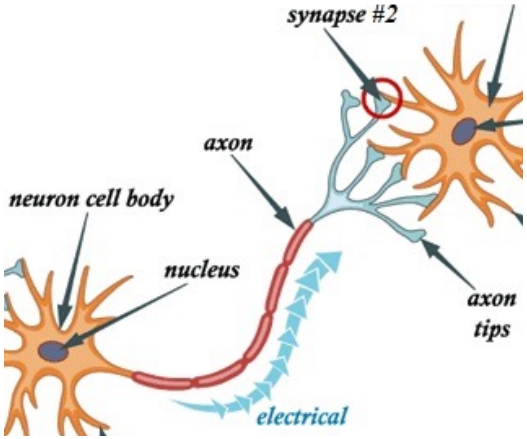


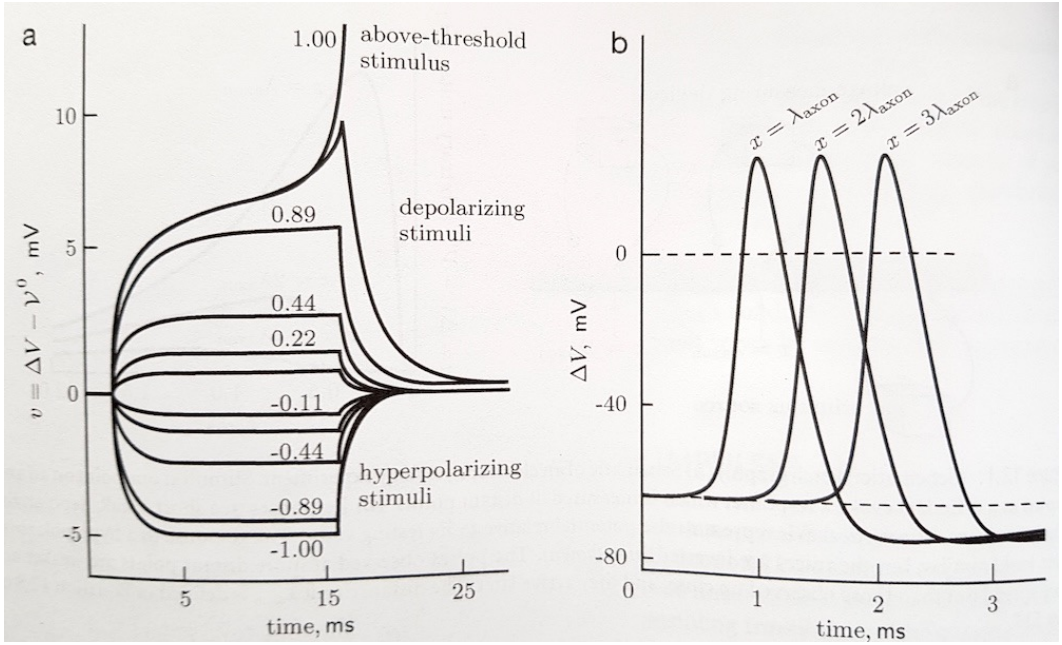
of an electrophysiology experiment. Stimuli at one point on an axo

- We are looking for propagating waves of $\Delta V - V^0$
- We only have a dissipative equation:

$$j_i = j_{p,i} - g_i \Delta \mu_i$$

Observations of action potential in axon





- When stimulated beyond a threshold the axon changes polarization for a short while
- This pulse travels along the axon at constant speed (0.1-120 m/s)
- Peak potential and pulse shape are independent of distance
- afterhyperpolarization
- harder to stimulate during a refractory period
- The peak and the shape is independent of the exact triggering pulse

Squid giant axon

Numerical example:

	$c_{i,o}$ [mM]	$c_{i,in}$ [mM]	V_i^N [mV]	g_i/g_{K+}
K^+	20	400	-75	1
Na^+	440	50	54	$\frac{1}{25}$ (* 8)
Cl^-	560	52	-59	$^1/_2$