Mid-term exam in FYS5120, 2013

Home Exam from monday the 24th of february until tuesday the 5th of march.

Answers to the problems may be written by hand or by word processing;- in english or a scandinavian language. There will be a box at the administative office of the Physics department where written answers may be delivered. The very last deadline for delivering written answers is wednesday the 6th of march, at the lecture at time 14.15. Write your candidate number (not name!) at the top of the front page of your written answer. Your candidate number is obtained from the student web.

* * *

In all the following problems we will consider massless QED (i.e. $m = m_0 = 0$). Furthermore, we will use the following ("general") photon propagator; - cfr. eq. (9.58) in Peskin and Schroeder (hereafter called "PS"):

$$D^{\mu\nu}(k) = \frac{(-g^{\mu\nu} + \eta \frac{k^\mu k^\nu}{k^2})}{(k^2 + i\epsilon)} \ , \tag{1}$$

where the choice $\eta = 0$ corresponds to the (most common) Feynman gauge. In the following you should calculate the three basic loop diagrams within dimensional regularization and give the result in terms of the dimension d, the external momentum p (or q), and Gamma-functions. During calculations you might need the formula

$$B(\alpha, \beta) = \int_0^1 dx\, x^{\alpha-1} (1-x)^{\beta-1} = \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha + \beta)} \ . \tag{2}$$

Problem 1

Calculate the expression $-i\Sigma(p)$ for the electron self energy;- cfr. the figure at page 217 in PS. Calculate separately the term corresponding to Feynman gauge ($\eta = 0$) and the term proportional to η in the propagator in eq. (1).

Problem 2

Calculate the expression $\Delta\Gamma^\mu(p', p)$ for the vertex correction (cfr. the figure at page 189 in PS) in the limit $q \to 0$ for the external photon momentum (i.e. $p' \to p$). Calculate separately the term corresponding to Feynman gauge ($\eta = 0$) and the term proportional to η.

Problem 3

Show by explicit derivation of $\Sigma(p)$ from **Problem 1** that

$$\frac{\partial \Sigma(p)}{\partial p_\mu} \sim \Delta\Gamma^\mu(p, p) \tag{3}$$

for both the two parts of Σ and $\Delta\Gamma^\mu$ (the ones for $\eta = 0$, and the ones proportional to η in eq. (1)).

Problem 4

Calculate the expression for the vacuum tensor $i\Pi^{\mu\nu}(q)$, as shown at page 245 in PS. Show explicitely that $\Pi^{\mu\nu}(q)$ within dimensional regularization has the form

$$\Pi^{\mu\nu}(q) = (g^{\mu\nu} q^2 - q^\mu q^\nu)\,\Pi(q^2) \ , \tag{4}$$

and determine $\Pi(q^2)$ in terms of Γ functions, q^2 and the dimension d.

Problem 5

Define the bare fine structure constant $\alpha_0 = (e_0)^2/(4\pi)$ and similarly $\alpha = e^2/(4\pi)$ for the renormalized fine structure constant. Define the the renormalized "running fine structure constant" $\alpha(\mu)$ at $q^2 = -\mu^2$ by

$$\alpha(\mu) = \alpha_0 \Big(1 + \Pi(q^2 = -\mu^2)\Big) \, . \tag{5}$$

Remember that Π is proportional to $(e_0)^2$. Use $\alpha_0 = \alpha + \mathcal{O}(\alpha^2)$, and show that

$$\mu \frac{\partial \alpha}{\partial \mu} = K \cdot (\alpha)^2 + \mathcal{O}(\alpha^4) \tag{6}$$

Determine the number K, and find (by neglecting the $\mathcal{O}(\alpha^4)$ terms) the relation between two different α's at two renormalization points μ_1 and μ_2.