

UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Relativistic quantum field theory (FYS5120 + FYS9210)

Day of exam: March 25, 2025

Exam hours: 4 hours

This examination paper consists of 4 pages.

(including the title page and the formula collection at the end)

Appendices: none

Permitted materials: 3 A4 pages (two-sided) with own notes.

Make sure that your copy of this examination paper is complete before answering.

Midterm exam

Autumn 2025: Advanced quantum field theory (FYS5120 + FYS9120)

↪ **Carefully read** all questions before you start to answer them! Note that you don't have to answer the questions in the order presented here, so start with those that you feel most sure about. Problems where slightly more involved/time-consuming calculations are needed are marked with an asterisk (*); you can fully answer subsequent questions even without actually solving those. Keep your descriptions self-contained, but as short and concise as possible – and generally try to only answer what is actually asked for (just for reference, the complete set of calculations and answers fits on 5 L^AT_EX pages)! Answers given in English are preferred; however, feel free to write in Norwegian/Swedish/Danish if you struggle with formulations!

Maximal number of available points: 40.

Good luck!

Problem 1

In the **path integral formalism**, the n -point correlation function for any type of field ϕ can be written as

$$\langle \Omega | T \{ \phi(x_1) \dots \phi(x_n) \} | \Omega \rangle = \frac{\int \mathcal{D}\phi \phi(x_1) \dots \phi(x_n) \exp \left[i \int d^4x \mathcal{L} \right]}{\int \mathcal{D}\phi \exp \left[i \int d^4x \mathcal{L} \right]}.$$

- a)* Use this formalism to derive the QED vertex rule, starting from the above expression (suitably adapted to this specific application). You can use all formulas stated in the Appendix of this exam without further explanation. (4 points)
[Hint: Try to keep this short, by using a smart notation and avoiding to write out small intermediate steps!]
- b) As an application of this formalism, we learned that one can simply ‘read off’ the vertex rules from a given interaction term in the Lagrangian. Draw the Feynman diagram and state the corresponding Feynman rules for each of the following interaction terms:

1. $\mathcal{L} \supset -c_1 A^2 A_\mu \bar{\psi} \gamma^5 \gamma^\mu \psi$
2. $\mathcal{L} \supset -ic_2 \{ \phi \phi^* \partial_\mu A^\mu + 2A^\mu \phi^* \partial_\mu \phi \}$
3. $\mathcal{L} \supset -c_3 (\partial_\mu \phi)^2 \phi$

Also state the dimension of the (real) coupling constant c_i in each case.

(6 points)

Problem 2

We now turn to aspects of the **standard model** of particle physics (SM).

- a) Consider a version of the SM where the local gauge symmetry group is the same, but where the Higgs field H is charged under $SU(3)_c \times U(1)_Y$ instead of $SU(2)_L \times U(1)_Y$. How many (real) components does H have in this case? Now assume that the potential $V(H)$ is of a form such that the symmetry is spontaneously broken as $SU(3)_c \times U(1)_Y \rightarrow SU(2) \times U(1)_{\text{em}}$. How many massive and how many massless gauge bosons does such a theory contain in total, and how many physical Higgs bosons? Demonstrate that the number of degrees of freedom before and after symmetry breaking is the same. (5 points)
- b) Draw the two dominant Feynman diagrams describing the decay of a Higgs particle to two gluons, $h \rightarrow gg$. Briefly explain your choice of diagrams. Argue that the same type of diagrams also gives an important contribution to the decay into two photons, $h \rightarrow \gamma\gamma$. Which other type of diagrams could potentially contribute significantly to the latter process (but not to the decay into gluons)? Estimate the relative importance of these two types of diagrams to the decay rate $\Gamma_{h \rightarrow \gamma\gamma}$ (5 points)

[Hint: You need to reflect about the relative size of couplings, as well as mass and/or energy scales.]

Problem 3

Let us consider the phenomenology of a **complex scalar field** Φ , (locally) charged under an (unbroken) Abelian gauge group $U(1)'$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)(D^\mu\Phi^*) - V(|\Phi|), \quad (1)$$

where $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $D_\mu = \partial_\mu - igB_\mu$, and the potential is an arbitrary function of $|\Phi|$; B^μ is the gauge field, and g the coupling strength of the $U(1)'$.

- a)* Write down the amplitude $i\mathcal{M}$ for the annihilation process $\phi\phi^* \rightarrow BB$, and simplify it as far as possible. Show explicitly that the result satisfies the Ward identity, i.e. that $\mathcal{M} = \mathcal{M}_{\mu\nu}\epsilon^\mu(k_1)^*\epsilon^\nu(k_2)^*$ vanishes when replacing either of the polarization vectors with the corresponding 4-momentum. (6 points)
- b) When squaring the amplitude and summing over all possible vector polarizations (which you are not supposed to do here), what is your *expectation* for the contributions from *i)* transverse polarization states and *ii)* all other polarization states? How would this expectation be modified in a non-Abelian gauge theory? (2 points)
- c) State explicitly the gauge transformations that leave the above Lagrangian invariant. Now assume that the potential is of such a form that spontaneous

symmetry breaking occurs, $\langle \Phi \rangle \neq 0$. Re-parameterize Φ in terms of two real scalar fields σ and π such that $\langle \sigma \rangle = 0$ and the Lagrangian remains manifestly gauge invariant. Write down this Lagrangian, and state explicitly the local (gauge) transformations that leave it invariant. (5 points)

- d)* Calculate the amplitude for the annihilation of two scalars σ into a pair of gauge bosons in the spontaneously broken theory. Choose for simplicity the unitary gauge to perform this calculation; in this case there is apparently only one diagram contributing at lowest order, coming from a term $\mathcal{L} \supset cA^2\sigma^2$. Compute $\sum_{\epsilon} |\mathcal{M}|^2$ by using the standard polarization sum for massive vector bosons, i.e. $\sum_{\epsilon} \epsilon^{\mu}(k)^* \epsilon^{\nu}(k)^* = -\eta^{\mu\nu} + k^{\mu}k^{\nu}/m_B^2$. Next, confirm that this is indeed the correct prescription by explicitly computing the individual amplitudes $\mathcal{M}_{\epsilon}$ for all physical polarizations ϵ , and then summing over $|\mathcal{M}_{\epsilon}|^2$. How would this comparison change if you had not chosen the unitary gauge? (7 points)

[*Hint: Remember that longitudinal polarizations take the form $\epsilon_{\mu}^L = (0, 0, 0, 1)$ in the rest frame of the particle. Then perform a Lorentz transformation of the two polarizations to the CMS frame, using $p^{\mu} = (E, \mathbf{p}) = \gamma m(1, \beta \hat{\mathbf{p}})$ for a massive particle with 3-momentum direction $\hat{\mathbf{p}}$.]*

- e) **Bonus question.** From the answer to the previous problem, you can conclude that the amplitude for longitudinal polarizations grows as $\mathcal{M} \propto s$ for large CMS energies $\sqrt{s}$. Since $|\mathcal{M}|^2$ is essentially a probability, such an unbounded growth with energy is very problematic (using the optical theorem, indeed, we will see in the lecture that such a scaling automatically violates unitarity). How is this problem solved? (Up to 5 bonus points)

Formulas that you might find useful:

- Free theory generating functionals:

$$\int \mathcal{D}A \exp \left[i \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J_{\mu} A^{\mu} \right) \right] \propto \exp \left[-\frac{1}{2} \int d^4x d^4y J_{\mu}(y) D_F^{\mu\nu}(x-y) J_{\nu}(x) \right] \quad (2)$$

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left[i \int d^4x (\bar{\psi} \not{\partial} \psi - m \bar{\psi} \psi + \bar{\eta} \psi + \bar{\psi} \eta) \right] \propto \exp \left[- \int d^4x d^4y \bar{\eta}(y) S_F(x-y) \eta(x) \right] \quad (3)$$