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http://www.uio.no/studier/emner/matnat/fys/FYS5120/v26/

Lecture spring 2026:

Quantum field theory II

Problem sheet 1

Problem 1

As a warm-up, to re-fresh our knowledge of canonical quantization, let us consider the Lagrangian for a free complex scalar field ϕ,

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 \,.$$

Compared to the real case, this Lagrangian exhibits an additional symmetry, $\phi \to \exp[i\alpha]\phi$, where α is an arbitrary phase.

a) Show that this is indeed a symmetry, and user Noether's theorem to demonstrate that the corresponding conserved charge is given by $Q \propto \int d^3x \, \Im(\dot{\phi}^* \phi)$.

b) Derive the Hamiltonian, and diagonalize it by introducing creation and annihilation operators.

c) Rewrite the conserved charge in terms of these annihilation and creation operators. How does the charge between the two types of particles differ?

Problem 2

As another warm-up, to re-fresh your hands-on experience with rate calculations, consider muon production in QED, $e^+e^- \to \mu^+\mu^-$, and calculate the total cross section σ (at tree-level) for this process.
[Hints: Recall that a muon is just a heavy 'cousin' of the electron (only considering QED, it would thus for example be stable for the same reason that an electron is stable). There is only one diagram that contributes to the process considered here.]

Problem 3

Following up on the theme of the previous exercise, let us consider Yukawa theory – i.e. a fermion interacting with a real scalar as follows:

$$\mathcal{L} = \overline{\psi}\left(i\partial\!\!\!/ - m_\psi\right)\psi + \frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right)^2 - \frac{1}{2}m^2\phi^2 - \lambda\phi\overline{\psi}\psi\,.$$

a) Calculate the decay rate $\Gamma_{\phi\to\psi\overline{\psi}}$.

b) Now calculate the decay rate $\Gamma_{\phi\to\psi_1\overline{\psi}_2}$ into two *different* fermions ψ_1 and ψ_2, with masses m_1 and m_2, that interact with the scalar as determined by $\mathcal{L}_{\text{int}} = -\lambda\phi\overline{\psi}_1\psi_2$ (you can thus use the same Feynman rule, $-i\lambda$, for the vertex).

c) Back to the original version (with only one scalar), calculate the differential cross section $d\sigma/dt$ for 'Bhabha scattering' in this theory, i.e. for the process $\psi\overline{\psi}\to\psi\overline{\psi}$. Express the result exclusively in terms of Mandelstam variables and compare it to the corresponding result in QED [as discussed in QFT1; you can also just look up the latter expression on Wikipedia ;)].
[Hint: Now there are two diagrams to consider; be careful with the relative sign!]