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http://www.uio.no/studier/emner/matnat/fys/FYS5120/v26/

Lecture spring 2026:

Quantum field theory II

Problem sheet 2

Problem 4

Consider a hermitian $N \times N$ matrix B and calculate the N-dimensional Gaussian integral

$$\int d^N p \, e^{-\frac{1}{2}p^*_m B_{mn} p_n + \frac{1}{2}J^*_n p_n + \frac{1}{2}J_n p^*_n} \, ,$$

where m, n run from 1 to N. How does the result generalize to the infinite-dimensional case? Finally, calculate these integrals also over Grassmann variables (i.e. use $p_i \to \theta_i$ with $\theta^i \theta^j = -\theta^j \theta^i$; and the same for J_i).
[Hint: Consider first the 1D integral, which you can solve by completing the square, and then using $\left[\int dp \exp(-\frac{1}{2}p^2)\right]^2 = 2\pi$*.]*

Problem 5

The generating functional for a free, real scalar field is proportional to

$$Z_0[J] \propto \exp\left[-\int d^4x \int d^4y \frac{1}{2} J(x) D^F(x-y) J(y)\right] \, ,$$

where D^F is the Fermi propagator.

a) Show this claim explicitly, starting from the definition

$$Z_0[J] \equiv \int \mathcal{D}\phi \exp\left[i \int d^4x \left(-\frac{1}{2}\phi(\Box + m^2 - i\epsilon)\phi + J\phi\right)\right] ,$$

and using the defining properties of Green's functions.

b) Using the functional formalism, confirm explicitly that the two-point correlation function $\langle 0|T\{\hat{\phi}(x)\hat{\phi}(y)\}|0\rangle$ equals $D_F(x-y)$.

c) Repeat the same exercise as in b), but this time in Fourier space.

Problem 6

Using the newly introduced path integral formalism, start from the QED Lagrangian and derive all Feynman rules from the corresponding generating functional!

Problem 7

To further train our skills in 'reading off' vertex rules, what are the vertex rules corresponding to the following interaction terms? (and what is the mass dimension of the respective operator ?)

a) $\mathcal{L} \supset \lambda_a \, (\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi)$

b) $\mathcal{L} \supset \lambda_b \, (\bar{\psi}\gamma^5\gamma^\mu\psi)\partial_\mu\phi$

c) $\mathcal{L} \supset \lambda_c \, F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi$, where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

d) $\mathcal{L} \supset \lambda_d \, \phi^2\partial_\mu(\bar{\psi}\gamma^\mu\psi)$