# UNIVERSITY OF OSLO

## Faculty of Mathematics and Natural Sciences

**Exam in:** Relativistic quantum field theory (FYS5120 + FYS9210)

**Day of exam:** March 18, 2026

**Exam hours:** 4 hours

**This examination paper consists of 4 pages.**
(including the title page and the formula collection at the end)

**Appendices:** none

**Permitted materials:** 3 A4 pages (two-sided) with own notes.

*Make sure that your copy of this examination paper is complete before answering.*

# Midterm exam

Spring 2026: Advanced quantum field theory (FYS5120 + FYS9120)

⇝ **Carefully read** *all questions before you start to answer them! Note that you don't have to answer the questions in the order presented here, so start with those that you feel most sure about. Problems where slightly more involved/time-consuming calculations are needed are marked with an asterisk (∗); you can fully answer subsequent questions even without actually solving those. Keep your descriptions self-contained, but as short and concise as possible – and generally try to only answer what is actually asked for (just for reference, the complete set of calculations and answers fits on 5 LaTeX pages)! Answers given in English are preferred; however, feel free to write in Norwegian/Swedish/Danish if you struggle with formulations!*
*Maximal number of available points:* **40**.

*Good luck!*

**Problem 1**

Based on the path integral formalism, we learned how to 'read off' **Feynman rules** directly from the Lagrangian.

a) Draw the Feynman diagram and state the corresponding Feynman rules for each of the following interaction terms. Also state the dimension of the (real) coupling constant $c_i$ in each case. (*6 points*)

1. $\mathcal{L} \supset -ic_1\, F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi\,, \quad \text{with } \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$
2. $\mathcal{L} \supset -c_2\, \phi^2 W_+^\mu W_{-\mu}$
3. $\mathcal{L} \supset -c_3\, (\partial_\mu \phi)^2 \phi^2$

b) In the gauge-fixing procedure following Faddeev and Popov, an additional term $\mathcal{L} \supset \bar{c}(\partial^\mu D_\mu)c$ appeared in the Lagrangian. Write out the covariant derivative for this case, and from there determine the two Feynman rules that are 'hidden' in this term. (*4 points*)

**Problem 2**

We now turn to aspects of the **standard model** of particle physics (SM).

a) State the full gauge group of the SM, and explain how the number of SM gauge bosons follows from the underlying *unbroken* symmetry. What is the total num- ber of bosonic (including scalar) degrees of freedom in the SM, before symmetry breaking? Then repeat the counting of bosonic degrees of freedom after spontaneous symmetry breaking, i.e. for the actually observed bosonic field content of the SM. What is the gauge group now? (*4 points*)

b)* Assume an electrically and color-neutral new scalar particle $\chi$ that dominantly couples to the SM via couplings to the top quark, i.e. $\mathcal{L}_{\text{int}} \supset -\lambda\chi\bar{t}t - \lambda_5\chi\bar{t}\gamma^5 t$. Draw one typical Feynman diagram that contributes to the process $\chi\chi \to \gamma\gamma$, i.e. the annihilation of two $\chi$ particles into a pair of photons. How many diagrams are there in total, for this process (at leading non-vanishing order)? How many additional diagrams would there be for the annihilation into *gluons*?
Assume now that the cross section $\sigma_{\chi\chi\to\gamma\gamma}$ has been measured in some experiment – what would you predict for the annihilation into gluons? In other words, calculate the ratio $\sigma_{\chi\chi\to gg}/\sigma_{\chi\chi\to\gamma\gamma}$ and provide a numerical estimate! For the sake of this estimate, you can neglect the effect of the additional diagram topologies that appear for $\chi\chi \to gg$. (*7 points*)

[*Hint: you don't have to fully calculate the individual* $|\mathcal{M}|^2$ *to obtain the result. Also the total number of diagrams does not affect the ratio (why?).* ]

**Problem 3**

In this problem we explore how the introduction of Goldstone bosons or **Stueckelberg fields** can restore the symmetry in the presence of massive gauge bosons.

a) Consider a massive Abelian gauge boson $B^\mu$ coupling to a Dirac fermion $\psi$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_B^2 B_\mu B^\mu + \overline{\psi}(i\not{D} - m_\psi)\psi\,, \tag{1}$$

with $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $D_\mu = \partial_\mu - igB_\mu$. What is the simplest way of changing this Lagrangian, by introducing an additional real scalar field $\phi$, such that the resulting theory features a local $U(1)$ symmetry? State those (infinitesimal) symmetry transformation explicitly. Can the field $\phi$ be massive ? (*4 points*)

b)* Write down the amplitude $i\mathcal{M} = \mathcal{M}_{\mu\nu}\epsilon^\mu(k_1)^*\epsilon^\nu(k_2)^*$ for the annihilation process $\psi\overline{\psi} \to BB$, at leading order in perturbation theory, and simplify it as far as possible (using the Dirac equation). Compute $\mathcal{M}_{\mu\nu}k_1^\mu$, again simplifying the result as much as possible. Discuss the result (or explain what you expect if you did not arrive at the final result) ! (*5 points*)
[*Hint: You need to use the Dirac equation, recalling that* $\not{p}$ *has eigenvalues with opposite signs when acting on fermions and anti-fermions, respectively.* ]

c) Now consider the non-Abelian version of the situation considered in a). In this case, a mass term for the vector fields cannot be made gauge-invariant by the introduction of a single real scalar field $\phi$. Instead, one defines the corresponding field $U(x) \equiv \exp\left(\frac{i}{v}\pi^a t^a\right)$, where $v$ is a constant of mass-dimension one and $t^a$ are the group generators (in some representation $r$). Now consider

$$\mathcal{L}_{\text{mass}} \equiv v^2 \text{Tr}\left[(D_\mu U)^\dagger (D^\mu U)\right]\,, \tag{2}$$

where $D_\mu U = \partial_\mu U - igB^a_\mu t^a U$. Show that $\mathcal{L}_{\rm mass}$ is gauge-invariant (hint for making this a very short task: what is the transformation expected for $U$, given the form of $D_\mu U$?), and derive the gauge boson mass $m_B$ that this term generates. The result depends on the representation of $U$; how is this determined in practice, in a given theory ? (*4 points*)

d)* The Lagrangian term from the previous problem also contains various couplings between the gauge bosons $B^a_\mu$ and the Goldstone fields $G^a \propto \pi^a$. Determine the proportionality factor, and derive the specific cubic interaction term that involves two Goldstone fields and one gauge boson. Does this term depend on the representation of $U$ ? (*6 points*)
[*Hint for d) and e): Make sure to only keep those terms in the expansion that you need to answer the question! Inside the trace, try to move all gauge fields to the left before the final simplification steps.*]

**Formulas that you might find useful:**

- Dirac algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{3}$$

- Properties of Lie algebra generators

$$(t^a)^\dagger = t^a; \qquad [t^a, t^b] = if^{abc}t^c; \qquad (t^a_G)_{bc} = -if^{abc} \tag{4}$$

- Casimir operators

$$t^a_r t^a_r \equiv \mathbf{1} \times C_2(r) = \mathbf{1} \times \begin{cases} \frac{N^2-1}{2N} & \text{for fundamental rep. of SU(N)} \\ N & \text{for adjoint rep. of SU(N)} \end{cases} \tag{5}$$

$$\mathrm{Tr}[t^a_r t^b_r] \equiv \delta^{ab} \times C(r) = \delta^{ab} \times \begin{cases} \frac{1}{2} & \text{for fundamental rep. of SU(N)} \\ N & \text{for adjoint rep. of SU(N)} \end{cases} \tag{6}$$