

Chapter 7

Sparticle phenomenology

In this chapter we discuss the phenomenology of supersymmetric models and how to search for low-energy particle realisations of supersymmetry in experiments. We begin by returning to supersymmetry breaking in order to define some reasonable and (partially) motivated subsets of the 124 MSSM parameters which can be used to define more constrained models. We then discuss supersymmetry at lepton and hadron colliders, and finally look at precision measurements that are indirectly sensitive to the existence of sparticles.

7.1 Models for supersymmetry breaking

Let us begin by taking a closer look at the models we use to motivate supersymmetry breaking, and what their phenomenological consequences are. This is important to keep in mind as most searches for supersymmetry are interpreted under certain assumptions on the supersymmetry breaking mechanism.

Generically such models can be illustrated as shown in Fig. 7.1. There is one or more **hidden sector** (HS) scalar superfield X – by hidden we mean that it has no or very small direct couplings to the MSSM fields – that has an *effective (non-renormalizable) coupling* to the MSSM scalar fields Φ_i of the form

$$\mathcal{L}_{\text{HS}} = -\frac{1}{M}(\bar{\theta}\theta)X\Phi_i\Phi_j\Phi_k, \quad (7.1)$$

where M is some large scale, *e.g.* the Planck scale, that suppresses the interaction. Figure 7.2 shows interactions that can lead to such terms, where M is the mass scale of some **mediator** particle Y .

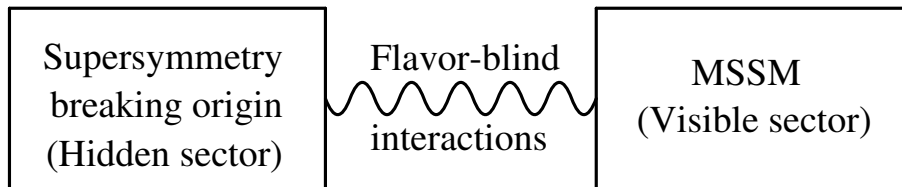


Figure 7.1: A generic illustration of how to generate soft breaking terms [18].

If the hidden sector is constructed so that X develops a vev for its auxillary F -component field, F_X ,

$$\langle X \rangle = \theta\theta\langle F_X \rangle, \quad (7.2)$$

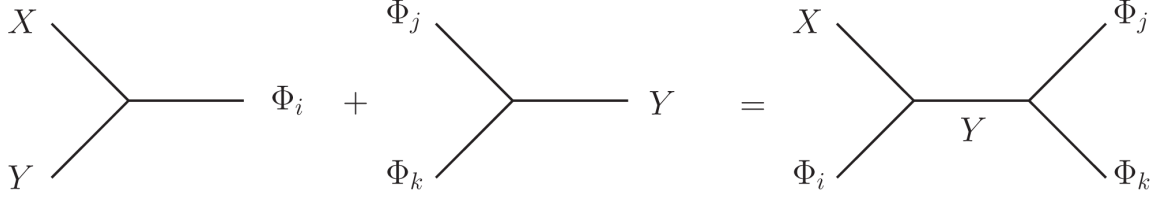


Figure 7.2: Interactions leading to effective 4-particle couplings in our example.

it breaks supersymmetry spontaneously, see the discussion leading up to Eq. (5.5). As a result, the interaction in (7.1) will produce a soft-term of the form of the second term in Eq. (5.9),

$$\mathcal{L}_{\text{soft}} = -\frac{\langle F_X \rangle}{M} A_i A_j A_k, \quad (7.3)$$

with the soft-mass parameter

$$m_{\text{soft}} = \frac{\langle F_X \rangle}{M}.$$

This has reasonable limits in that $m_{\text{soft}} \rightarrow 0$ as $\langle F_X \rangle \rightarrow 0$, which is the limit of no supersymmetry breaking, and $m_{\text{soft}} \rightarrow 0$ as $M \rightarrow \infty$, where the interaction with the hidden sector is decoupled because the mediating particle Y becomes too heavy to have any influence.

We will now look at two possible ways to construct such a hidden sector called Planck-scale Mediated Supersymmetry Breaking (PMSB) and Gauge Mediated Supersymmetry Breaking (GMSB).

7.1.1 Planck-scale Mediated Supersymmetry Breaking (PMSB)

In Planck-scale mediated supersymmetry breaking (PMSB) we blame some gravity mechanism for mediating the supersymmetry breaking from the hidden sector to the MSSM so that the scale of the breaking is $M = M_P = 2.4 \cdot 10^{18}$ GeV. Then we need to have $\sqrt{\langle F_X \rangle} \sim 10^{10} - 10^{11}$ GeV in order to get $m_{\text{soft}} \simeq 50 - 5000$ GeV, which is roughly of the right magnitude not to re-introduce the hierarchy problem. The use of $\sqrt{\langle F_X \rangle}$ is just a conventional shorthand notation for the magnitude of the vev of whichever F -term that breaks supersymmetry. This is called the **supersymmetry breaking scale**.

The complete soft terms for such a mechanism can then be shown to be

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & -\frac{\langle F_X \rangle}{M_P} \left(\frac{1}{2} f_i \lambda_i^a \lambda_i^a + \frac{1}{6} y'_{ijk} A_i A_j A_k + \frac{1}{2} \mu'_{ij} A_i A_j + \frac{\langle F_X \rangle^*}{M_P^2} x_{ijk} A_i^* A_j A_k + \text{c.c.} \right) \\ & - \frac{|\langle F_X \rangle|^2}{M_P^2} k_{ij} A_i A_j^*. \end{aligned} \quad (7.4)$$

Incidentally, we can now see why we assumed the maybe-soft breaking terms to be unimportant, as in this model they are suppressed by $|\langle F_X \rangle|/M_P^2$ compared to the other masses.

If one assumes a minimal form for the parameters at the GUT scale, motivated by the wish for unification, *e.g.* $f = f_i$, $y'_{ijk} = \alpha y_{ijk}$, $\mu'_{ij} = \beta \mu$, and $k_{ij} = k \delta_{ij}$, then all the soft terms

are fixed by just four parameters

$$m_{1/2} = f \frac{\langle F_X \rangle}{M_P}, \quad m_0^2 = k \frac{|\langle F_X \rangle|^2}{M_P^2}, \quad A_0 = \alpha \frac{\langle F_X \rangle}{M_P}, \quad B_0 = \beta \frac{\langle F_X \rangle}{M_P}.$$

The resulting phenomenology is called **minimal supergravity**, or **mSUGRA/CMSSM**. This is minimal in the sense of the form of the parameters, and is the most studied, but perhaps not best motivated, version of the MSSM. Usually, B_0 and $|\mu|$ are exchanged for $\tan \beta$ at low scales using the EWSB condition in Eq. (6.31), so it is common to say that there are four and a half parameters in the model: $m_{1/2}$, m_0 , A_0 , $\tan \beta$ and $\text{sgn } \mu$.

7.1.2 Gauge Mediated Supersymmetry Breaking (GMSB)

An alternative to PMSB is gauge-mediated supersymmetry breaking where soft terms come from *loop diagrams* with **messenger** superfields that get their own mass by coupling to the hidden sector supersymmetry breaking vev, and that have Standard Model gauge interactions. By dimensional analysis we must have

$$m_{\text{soft}} = \frac{\alpha_i}{4\pi} \frac{\langle F \rangle}{M}.$$

If now the supersymmetry breaking scale $\sqrt{\langle F \rangle}$ and the messenger mass M are roughly comparable in size, which is reasonable given where the messenger mass comes from, then $\sqrt{\langle F \rangle} \simeq 100 \text{ TeV}$ can give a viable sparticle spectrum. Notice that there is now a lot less RGE running for the parameters in the GMSB compared to the PMSB since the soft masses are created at a rather low scale.

One way of thinking about how these mass terms appear is that the messenger field(s) get masses from hidden sector vevs and contribute to for example gaugino mass terms through diagrams such as the one in Fig. ??, where messenger scalars and fermions run in the loop, and their masses from the hidden sector vevs are symbolised by the mass insertions. Note that scalars can only get mass contributions like this at two-loop order since the messenger interaction is a gauge interaction, involving gauge bosons or gauginos in the MSSM. In order not to spoil GUT unification messengers are often assumed to have small mass splittings and come in N_5 complete $\mathbf{5} + \bar{\mathbf{5}}$ (fundamental) representations of $SU(5)$.

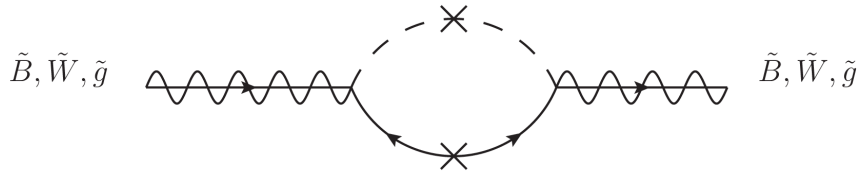


Figure 7.3: Diagram for GMSB giving masses to the gauginos. The messenger scalars and fermions run in the loop.

The minimal parametrisation of GMSB models is in terms of the scale $\Lambda = \frac{\langle F \rangle}{M}$, the messenger mass M , the number of representations N_5 , and $\tan \beta$ for the EWSB criterion

(instead of μ and b). This gives the soft masses

$$M_i = \frac{\alpha_i}{4\pi} \Lambda N_5, \quad (\text{gaugino masses}) \quad (7.5)$$

$$m_j^2 = 2\Lambda^2 N_5 \sum C(R)_i \left(\frac{\alpha_i}{4\pi} \right)^2, \quad (\text{scalar masses}) \quad (7.6)$$

where the sum is over the quadratic Casimir invariant for the scalar superfield Φ_j that the scalar field belongs to. We clearly see that the scalar soft-masses are a two-loop effect as discussed above.

While this parameterisation looks independent of M , the messenger scale sets the starting point of the RGE running of the sparticle masses, and thus influences their magnitude. For example, the tri-linear soft-term couplings a_{ijk} are expected to be very small at the messenger scale, and are effectively set to zero, however, due to RGE running they are small, but non-zero at the electroweak scale. Since the scalar masses m_j scale as $\sqrt{N_5}$ compared to N_5 for the gauginos, we in general expect the scalars to be lighter in GMSB models. One should also notice that this parameterisation gives the same hierarchy of gaugino masses as in mSUGRA, $M_3 > M_2 > M_1$, since (7.5) is ordered in terms of the strength of the gauge couplings α_i . The origin of the hierarchy is different since in mSUGRA it comes from the running of the parameters down from the GUT scale.

7.2 Supersymmetry at lepton colliders

Most lepton colliders are e^+e^- -colliders, although plans are being made for a muon collider where there is less bremsstrahlung because of the higher muon mass, meaning that higher energies can be reached. The highest energy so-far at an e^+e^- -collider was 209 GeV CoM-energy at LEP2 in 2000.

Most supersymmetry searches at lepton colliders rely on pair production from $e^+e^- \rightarrow \gamma^*/Z^*$ to set limits, and for R-parity conserving supersymmetry we (again) rely on missing energy $\cancel{E}$ as an essential signature, however, since the longitudinal momentum is now exactly known full energy conservation can in principle be used. In practice this is challenging at high energies because of collinear Bremsstrahlung. This will be a particularly difficult for a future 0.5–3.0 TeV CoM International Linear Collider (ILC) or the Compact Linear Collider (CLIC) project.¹

We can estimate the amplitude of the sfermion pair production process shown in Fig. 7.18. We can write down the matrix element as:

$$\mathcal{M} = \bar{v}ie\gamma^\mu u \frac{-ig_{\mu\nu}}{k^2 + i\epsilon} [-ie \cdot e_f(p_1 - p_2)^\nu], \quad (7.7)$$

which gives a squared matrix element of, assuming that the CoM s is much greater than m_Z and taking into account both the photon and the Z :

$$|\mathcal{M}|^2 \simeq \frac{g^4 e_f^2}{8 \cos \theta_W} \frac{st + (m_f^2 - t)^2}{s^2} \times (1 + (4 \sin^2 \theta_W - 1)^2). \quad (7.8)$$

¹For more information on these projects see the websites for the International Linear Collider <http://www.linearcollider.org/> and the Compact Linear Collider <http://cllc-study.org/>