

Chapter 5

Breaking supersymmetry

In Chapter 2 we saw that supersymmetry predicts scalar partner particles with the same mass as the known fermions, and new fermions with the same mass as the known vectors, living in the same irreducible representation. These, somewhat unfortunately, contradict experiment by not existing. In this chapter we will look at how we can break supersymmetry so that the new states become more massive, and avoid current experimental bounds. This discussion will be held at a general level, and concrete realisations of this breaking will have to wait until Chapter 6.

5.1 Spontaneous supersymmetry breaking

In the Standard Model we have already faced this problem: the vector bosons should remain massless under the gauge symmetry of the model. Yet, they are observed to be very massive. This is solved with the introduction of the Higgs mechanism and **spontaneous symmetry breaking** in the **scalar potential**.¹ The idea is that while there is a symmetry of the Lagrangian – in the Standard Model the gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$ – this may not be a symmetry for the particular vacuum state of the theory (the lowest energy state), thereby allowing the properties of the vacuum to supply the masses. In the Standard Model this is achieved by the shape of the scalar potential having a minimum for a non-zero value of the Higgs field which means that the value of the field there – its vacuum expectation value – can contribute to the masses. Would it not be great if we could have a similar *spontaneous symmetry breaking* in order to break supersymmetry this way and boost the masses of supersymmetric particles beyond current limits?

To find the properties of the vacuum state for our supersymmetric models we start by pointing out that we can write the supersymmetric Hamiltonian as

$$H = \frac{1}{4}(Q_1\bar{Q}_1 + \bar{Q}_1Q_1 + Q_2\bar{Q}_2 + \bar{Q}_2Q_2).$$

To see this, consider first

$$\begin{aligned} \{Q_A, \bar{Q}_{\dot{B}}\}\bar{\sigma}^{\nu\dot{B}A} &= 2\sigma^\mu_{A\dot{B}}\bar{\sigma}^{\nu\dot{B}A}P_\mu \\ &= 2\text{Tr}[\sigma^\mu\bar{\sigma}^\nu]P_\mu \\ &= 4g^{\mu\nu}P_\mu = 4P^\nu. \end{aligned}$$

¹The potential of the Lagrangian are those terms not containing derivatives of the fields (kinetic terms). The scalar potential are such terms that contain only scalar fields.

Now,

$$\begin{aligned} H &= P^0 = \frac{1}{4}\{Q_A, \bar{Q}_{\dot{B}}\}\bar{\sigma}^{0\dot{B}A} \\ &= \frac{1}{4}(Q_1\bar{Q}_{\dot{1}} + \bar{Q}_{\dot{1}}Q_1 + Q_2\bar{Q}_{\dot{2}} + \bar{Q}_{\dot{2}}Q_2). \end{aligned}$$

As discussed in Section 2.4 we have $Q_A^\dagger = \bar{Q}_{\dot{A}}$. Thus the Hamiltonian is **semipositive definite**, *i.e.* $\langle\Psi|H|\Psi\rangle \geq 0$ for any state $|\Psi\rangle$, so the energy of any state is always non-negative.

Imagine now that there exists some lowest lying state (or possibly degenerate states), the ground state(s) $|0\rangle$, that has vanishing energy $\langle 0|H|0\rangle = 0$. These states are supersymmetric – meaning invariant under the supersymmetry transformation – since, to fulfil the energy assumption, we must have

$$Q_A|0\rangle = \bar{Q}_{\dot{A}}|0\rangle = 0 \quad \text{for } \forall A, \dot{A}, \quad (5.1)$$

and are thus invariant under the supersymmetry transformations given by (4.3)

$$\delta_S|0\rangle = (\alpha^A Q_A + \bar{\alpha}_{\dot{A}} \bar{Q}^{\dot{A}})|0\rangle = 0. \quad (5.2)$$

The vanishing energy means that at this supersymmetric minimum of the potential the scalar potential must contribute zero, $V(A, A^*) = 0$, and thus, from Eq. (4.21),

$$\left\langle \frac{\partial W}{\partial A_i} \right\rangle = 0 \quad \text{and} \quad \langle A_i^* T_{ij}^a A_j \rangle = 0.$$

Conversely, if the scalar potential does contribute to the energy for the vacuum (ground state) $|0\rangle$, meaning that either

$$\left\langle \frac{\partial W}{\partial A_i} \right\rangle \neq 0 \quad \text{or} \quad \langle A_i^* T_{ij}^a A_j \rangle \neq 0,$$

in the minimum of the potential for some A_i , then *supersymmetry must be broken*. As in the Standard Model, the Lagrangian is still (super)symmetric, but $|0\rangle$ is not because (5.1) can no longer hold for all the Q s.

The **O’Raifeartaigh model** (1975) [8] is an example of a model that spontaneously breaks supersymmetry with three scalar superfields X, Y, Z , and the superpotential

$$W = \lambda Y Z + g X (Z^2 - m^2), \quad (5.3)$$

where λ, g and m are real non-zero parameters. The scalar potential is

$$\begin{aligned} V(A, A^*) &= \left| \frac{\partial W}{\partial A_X} \right|^2 + \left| \frac{\partial W}{\partial A_Y} \right|^2 + \left| \frac{\partial W}{\partial A_Z} \right|^2 \\ &= |g(A_Z^2 - m^2)|^2 + |\lambda A_Z|^2 + |\lambda A_Y + 2g A_X A_Z|^2, \end{aligned} \quad (5.4)$$

which can never be zero because setting $A_Z = 0$, which is needed for the second term, gives a non-zero contribution $g^2 m^4$ from the first term. Since the expectation value at the minimum that breaks supersymmetry is $\langle 0 | \frac{\partial W_i}{\partial A_i} | 0 \rangle$, and $F_i = -\frac{\partial W^*}{\partial A_i}$, the condition for spontaneous supersymmetry breaking with the O’Raifeartaigh mechanism can be written

$$\langle F_i \rangle \equiv \langle 0 | F_i(x) | 0 \rangle > 0, \quad (5.5)$$

hence it is often given the more generic name **F-term breaking** outside of the specific O’Raifeartaigh model. In general F -term breaking it is the **vacuum expectation value** (**vev**) of the auxiliary field of a scalar superfield that supplies the breaking.

In a gauge theory, a similar mechanism is found by adding the Fayet-Iliopolous term $\mathcal{L}_{FI} \sim 2kV$ where V is an abelian vector superfield. The vev of the $D(x)$ auxiliary field, $\langle D \rangle = \langle 0|D(x)|0 \rangle$, will then create a non-zero scalar potential.² This is called the **Fayet-Iliopolous model**, or **D-term breaking**.

5.2 Supertrace

Unfortunately, the above does not work in practice if all particles are at a low energy scale. The problem is that at *tree level* the **supertrace**, STr , of the mass matrix $\mathcal{M}$, meaning the weighted sum of eigenvalues of the mass matrix, can be shown to vanish, $\text{STr } \mathcal{M}^2 = 0$ even after spontaneous supersymmetry breaking.³

Definition: The **supertrace** is given by

$$\text{STr } (\mathcal{M}^2) \equiv \sum_s (-1)^{2s} (2s+1) \text{Tr}[M_s^2], \quad (5.6)$$

where $\mathcal{M}$ is the complete mass matrix of the Lagrangian, s is the spin of particles and M_s is the mass matrix of all spin- s particles.

For a theory with only scalar superfields, with two fermionic and two bosonic degrees of freedom each, and with mass matrices $M_{1/2}$ and M_0 containing the masses of each particle on the diagonal, this means that $\text{STr } (\mathcal{M}^2) = \text{Tr}[M_0^2 - 2M_{1/2}^2] = 0$. While before spontaneous supersymmetry breaking each fermion and boson in the same superfield had equal masses, we now have that the sum of (the square of) the fermion and boson masses over all the superfields is the same. The crucial consequence is that not all the scalar partners can be heavier than our known fermions in order to balance out the supertrace relationship.

Adding vector superfields does not help because the fermions there have spin-1 vector bosons partners, and in the supertrace these contributions get opposite sign, and cancel each other out. This relationship is a tree level relationship, meaning it does not take into account quantum corrections. However, accounting for those and using strong couplings to move the supertrace value away from zero, the effect does not seem to be enough to build a consistent model.

The solution to the supertrace dilemma is to put the spontaneous supersymmetry breaking up at some higher energy scale $\sqrt{\langle F \rangle}$ – significantly higher than the electroweak scale – that we do not currently have experimental access to, so that there are also superfields with fermion and boson partners with much higher masses that fulfil the supertrace relationship, and the electroweak scale fermions and bosons become only a small perturbation on it.

²It is *always* the auxiliary fields’ fault!

³See Ferrara, Girardello and Palumbo (1979) [\[9\]](#).

5.3 Soft breaking

There are many alternatives in the literature of how we can break supersymmetry spontaneously at a high energy scale. The names of some popular examples that we will return to are:

- Planck-scale Mediated Symmetry Breaking (PMSB)
- Gauge Mediated Symmetry Breaking (GMSB)
- Anomaly Mediated Symmetry Breaking (AMSB)

Common to all of them is that the mass scale of the particles in the supersymmetry breaking fields is so high, that we can not observe their effects directly. What we can do instead is to parameterise our ignorance by adding effective explicit supersymmetry breaking terms to our low-energy Lagrangian.

However, we cannot simply add arbitrary terms to the Lagrangian. The terms we can add are so-called **soft terms** with couplings of mass dimension one or higher. The disallowed terms with smaller mass dimension are terms that can lead to divergences in loop contributions to scalar masses (such as the Higgs) that are quadratic or worse (because of the high dimensionality of the fields in the loops). This will re-introduce the hierarchy problem.

The allowed terms are in superfield notation as follows:

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -\frac{1}{4T(R)}M\theta\theta\bar{\theta}\bar{\theta}\text{Tr}\{W^AW_A\} - \frac{1}{6}a_{ijk}\theta\theta\bar{\theta}\bar{\theta}\Phi_i\Phi_j\Phi_k \\ & -\frac{1}{2}b_{ij}\theta\theta\bar{\theta}\bar{\theta}\Phi_i\Phi_j - t_i\theta\theta\bar{\theta}\bar{\theta}\Phi_i + \text{h.c.} \\ & -m_{ij}^2\theta\theta\bar{\theta}\bar{\theta}\Phi_i^\dagger\Phi_j,\end{aligned}\tag{5.7}$$

where $M, a_{ijk}, b_{ij}, t_i \in \mathbb{C}$ and $m_{ij}^2 \in \mathbb{R}$ are the couplings. Note that these terms are explicitly *not* supersymmetric. From the $\theta\theta\bar{\theta}\bar{\theta}$ -factors we see that only the lowest order in θ component fields of the superfields contribute.

There are also some terms that are called "maybe-soft" terms:

$$\mathcal{L}_{\text{maybe}} = -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}c_{ijk}\Phi_i^\dagger\Phi_j\Phi_k + \text{h.c.}\tag{5.8}$$

with $c_{ijk} \in \mathbb{C}$. This last – oft ignored – type of term is soft as long as none of the scalar superfields is a singlet under all gauge symmetries. It is, however, quite difficult to get large values for c_{ijk} with spontaneous supersymmetry breaking. In the above terms we have not specified any gauge symmetry, which will, in the same way as it did for the superpotential, severely restrict the allowed terms. However, it turns out that soft-terms are responsible for most of the parameters in supersymmetric theories!

We can instead write the soft terms in terms of their component fields as

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -\frac{1}{2}M\lambda^A\lambda_A - \left(\frac{1}{6}a_{ijk}A_iA_jA_k + \frac{1}{2}b_{ij}A_iA_j + t_iA_i + \frac{1}{2}c_{ijk}A_i^*A_jA_k + \text{c.c.}\right) \\ & -m_{ij}^2A_i^*A_j.\end{aligned}$$

Here we clearly see that the m_{ij} couplings provide extra mass terms to the scalar partners A_i of the fermions in the scalar superfields, and M to the fermion partners λ_A of the vectors in

the vector superfields. Thus, they split the masses of the partners as we wanted to achieve with breaking supersymmetry.

Now, what happens to the hierarchy problem? Restricting ourselves to soft supersymmetry breaking terms guarantees that we end up with contributions to the Higgs mass of at most

$$\Delta m_h^2 = -\frac{\lambda_s}{16\pi^2} m_s^2 \ln \frac{\Lambda_{UV}^2}{m_s^2} + \dots, \quad (5.9)$$

at the leading order in Λ_{UV} , where m_s is the mass scale of the soft term. The correction to the Higgs mass is thus proportional to the masses of the supersymmetric particles.

This is the most important argument in favour of supersymmetry existing at low energy scales where we can detect it, because m_s can not be too large if we want the above corrections to be reasonably small. This is called the **little hierarchy problem**, and numerically, given that $16\pi^2 \sim 100$, and the couplings λ_s are not expected to be above unity, this means that we want $m_s \sim \mathcal{O}(1 \text{ TeV})$ in order to keep the cancellations reasonable and of the order of the measured Higgs mass, with little fine-tuning.

5.4 Exercises

Exercise 5.1

Show that adding a Fayet-Iliopolous term to the SQED constructed in Exercise [4.3](#) will break supersymmetry spontaneously.

Exercise 5.2

Write down the supertrace relationship for a spontaneously broken SQED.