

Chapter 6

The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is a minimal supersymmetric model in the sense that it has the smallest field (and gauge) content consistent with the known Standard Model fields. We will now construct this model on the basis of what we have learnt the previous chapters, and look at some of its consequences.

6.1 MSSM field content

To specify a supersymmetric model we need to specify the superfield content and the gauge symmetries. The gauge symmetry is already given as the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$. Previously we learnt that each (left-handed) scalar superfield S has a (left-handed) Weyl spinor ψ_A , a complex scalar A_s and an auxiliary complex scalar field F_s , since they are a $j = 0$ representation of the superalgebra. After an application of the equations of motion ψ_A and A_s have two fermionic and two bosonic degree of freedom remaining respectively, while the auxiliary field has been eliminated along with two fermionic d.o.f..

In order to construct a Dirac fermion, which are plentiful in the Standard Model, we need a right-handed Weyl spinor as well. We can acquire the needed right-handed Weyl spinor from the hermitian conjugate $\bar{T}^\dagger$ of a different scalar superfield $\bar{T}$ with the right-handed Weyl spinor $\bar{\varphi}_{\dot{A}}$.^[1] With these four fermionic d.o.f. we can construct *two* Dirac fermions, a particle–anti-particle pair, and four scalars, two particle–anti-particle pairs.

We use these two superfield ingredients to construct all the known fermions:

- To represent the Standard Model leptons we introduce the superfields l_i and $\bar{E}_i$ for the charged leptons (i is the generation index) and ν_i for the neutrinos, and form $SU(2)_L$ doublet superfield vectors $L_i = (\nu_i, l_i)$, which contain the left-handed Weyl spinors for the particles. We do not introduce the $\bar{N}_i$ that would contain right-handed neutrino spinors needed for massive Dirac neutrinos.^[2] This is a convention – the MSSM is older than the discovery of neutrino mass – and including $\bar{N}_i$ fields has some interesting

¹The bar here is used to (not) confuse us, it is part of the name of the superfields and does not denote any hermitian or complex conjugate. The bar signifies that this is the field where, when hermitian conjugated, we will pick the right-handed (bared) Weyl-spinor from in the Dirac fermion.

²The anti-neutrino contained in the superfield $\nu_i^\dagger$ is right-handed consistent with experiment.

consequences.^[3] The right-handed neutrinos themselves would not couple to anything since they are SM singlets,^[4] however, their supersymmetric scalar partners could be a potential dark matter candidate.

- For quarks the situation is similar. Up-type and down-type quarks get the superfields u_i , $\bar{U}_i$ and d_i , $\bar{D}_i$, forming the $SU(2)_L$ doublets $Q_i = (u_i, d_i)$.^[5]

With all possible apologies, we now change notation for the component fields to what is conventional in phenomenology, as opposed to pure theory. The left-handed Weyl spinors in these superfields will now be named for example l_{iL} for the spinor in the superfield l_i , and $\bar{l}_{iL}$ for the one in $\bar{E}_i$, and similarly for the other superfields. This signifies that they are the left-handed Weyl spinors for the particle and anti-particle, respectively. For the right-handed Weyl spinors in the hermitian conjugate superfields we swap the bar and L/R labels, so that for example $l_i^\dagger$ contains $\bar{l}_{iR}$ and $\bar{E}_i^\dagger$ contains l_{iR} .

The scalar component fields are named after the fermions using the tilde notation, for example in the superfield l_i we have the scalar $\tilde{l}_{iL}$ as the supersymmetric partner particle, often just called **sparticle**, of the fermion l_{iL} . Similarly, $\bar{E}_i$ contains $\tilde{l}_{iL}^*$ which is the anti-particle of $\tilde{l}_{iL}$, $l_i^\dagger$ contains $\tilde{l}_{iR}^*$, and $\bar{E}_i^\dagger$ contains $\tilde{l}_{iR}$. Since scalars do not have any notion of handedness the L or R here is just part of the conventional name, we still call these particles for left-handed and right-handed scalar leptons though.

Additionally, we need vector superfields, which, after the e.o.m. have eliminated the auxiliary field, contain a *massless* vector boson component field with two scalar d.o.f. and two Weyl-spinors, one of each handedness λ and $\bar{\lambda}$, with two fermionic degrees of freedom. Together these form a $j = \frac{1}{2}$ representation of the superalgebra. If the vector superfield is neutral, the fermions can form a Majorana fermion, if not they can be combined with the Weyl-spinors from other fields to form Dirac fermions.

Looking at the construction $V \equiv qt^a V^a$ for the vector superfields in the supersymmetric Lagrangian we see that, as expected, we need one superfield V^a per generator t^a of the algebra, with the normal $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ vector bosons as component fields. We call these superfields C^a , W^a and B^0 .^[6] In order to be really confusing, we use the following symbols for the fermions constructed from the respective Weyl-spinors: $\tilde{g}$, $\tilde{W}^0$ and $\tilde{B}^0$. The tilde here is supposed to tell us – just as for the scalar component fields of the scalar superfields – that they are supersymmetric partners of the known Standard Model particles.

We also need superfields for the scalar Higgs boson. Now life gets interesting. The Higgs $SU(2)_L$ doublet scalar field H used in the Standard Model cannot give mass to all fermions because it relies on the construction $H^C \equiv -i(H^\dagger \sigma_2)^T$ to give masses to up-type quarks, and possibly neutrinos. The superfield version of this cannot appear in the superpotential because it would mix left- and right-handed superfields. The minimal Higgs content we can get away with are two Higgs superfield $SU(2)_L$ doublets, which we will call H_u and H_d , indexing the quarks they give mass to.^[7] These must have (more on the reason for that in Sec. [6.2]) weak

³Note that component fields in the same superfield must have the same charge under all the gauge groups, *i.e.* the scalar partner of the electron has electric charge $-e$, so it cannot be a neutrino.

⁴They can not be colour-charged, they are right-handed singlets under $SU(2)_L$ thus they have zero weak isospin, but since they should also have zero electric charge the hypercharge must also be zero.

⁵Here we should really also include a color index a so that u_i^a is a component in an $SU(3)_C$ triplet superfield vector. We omit these for simplicity.

⁶And there we have another W.

⁷In some further insanity some authors prefer H_1 and H_2 so that you have no idea which is which.

hypercharge $Y = \pm 1$ for H_u and H_d respectively, so that we have the superfield doublets:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad (6.1)$$

where we have given the electric charges of the scalar superfield components of the superfield doublets based on the standard $Q = \frac{1}{2}Y + T_3$ relationship after electroweak symmetry breaking, where T_3 is the weak isospin. The fermion component fields of these superfields will be denoted $\tilde{H}_u^+$, $\tilde{H}_u^0$, $\tilde{H}_d^0$, and $\tilde{H}_d^-$. The scalar component fields of these fields, before their mixing following electroweak symmetry breaking, will have the same symbols as the superfields. (Yes, really!)

6.2 The kinetic terms

It is now straight forward to write down the kinetic terms of the MSSM Lagrangian giving the matter-gauge interaction terms

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & L_i^\dagger e^{\frac{1}{2}g\sigma W - \frac{1}{2}g'B} L_i + Q_i^\dagger e^{\frac{1}{2}g_s\lambda C + \frac{1}{2}g\sigma W + \frac{1}{3}\cdot\frac{1}{2}g'B} Q_i \\ & + \bar{U}_i^\dagger e^{\frac{1}{2}g_s\lambda C - \frac{4}{3}\cdot\frac{1}{2}g'B} \bar{U}_i + \bar{D}_i^\dagger e^{\frac{1}{2}g_s\lambda C + \frac{2}{3}\cdot\frac{1}{2}g'B} \bar{D}_i \\ & + \bar{E}_i^\dagger e^{2\frac{1}{2}g'B} \bar{E}_i + H_u^\dagger e^{\frac{1}{2}g\sigma W + \frac{1}{2}g'B} H_u + H_d^\dagger e^{\frac{1}{2}g\sigma W - \frac{1}{2}g'B} H_d, \end{aligned} \quad (6.2)$$

where g' , g and g_s are the couplings constants (strengths) of $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively, and $\frac{1}{2}\sigma_i$ and $\frac{1}{2}\lambda_a$ are the generators of $SU(2)_L$ and $SU(3)_C$. As a convention we assign the charges under $U(1)$, hypercharge, in units of $\frac{1}{2}g'$. All non-singlets of $SU(2)_L$ and $SU(3)_C$ have the same charge, the factor $\frac{1}{2}$ here is used to get by without accumulation of numerical factors since the algebras for the Pauli (σ_i) and Gell-Mann matrices (λ_a) are:

$$[\frac{1}{2}\sigma_i, \frac{1}{2}\sigma_j] = i\epsilon_{ijk}\frac{1}{2}\sigma_k,$$

and

$$[\frac{1}{2}\lambda_a, \frac{1}{2}\lambda_b] = if_{ab}^c\frac{1}{2}\lambda_c.$$

These conventions lead to the wanted Standard Model gauge transformations for the component fields and the familiar relations after electroweak symmetry breaking, $e = g \sin \theta_W = g' \cos \theta_W$.

We mentioned earlier that the two Higgs superfields have opposite hypercharge. This is needed for so-called **anomaly cancellation** in the MSSM. Gauge anomaly is the possibility that at loop level contributions to processes such as in Fig. [6.1](#) break the gauge invariance that we have established at the classical level in the Lagrangian, and ruins the predictability of the theory. This rather miraculously does not happen in the SM because it has the exactly field content it has, so that all such possible gauge anomalies exactly cancel – we do not know of a deeper reason for why it has exactly this field content. If we have only *one* Higgs doublet this cancellation does not happen for the MSSM. With two Higgs doublets, and with opposite hypercharge, it does.

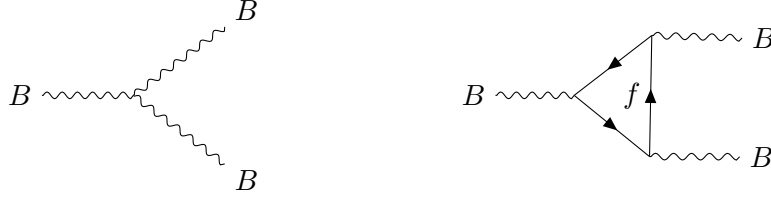


Figure 6.1: The tree level coupling between three gauge bosons B (left), and the one-loop fermion contribution to the same process (right).

6.3 Gauge terms

The pure gauge terms with supersymmetric field strengths are also fairly easy to write down:

$$\mathcal{L}_V = \bar{\theta}\bar{\theta} \text{Tr}[C^A C_A] + \bar{\theta}\bar{\theta} \text{Tr}[W^A W_A] + \frac{1}{2} \bar{\theta}\bar{\theta} B^A B_A, \quad (6.3)$$

where we have used the Dynkin indices of the gauge group representations

$$T(R)_L = \text{Tr} \left[\frac{1}{2} \sigma_1 \cdot \frac{1}{2} \sigma_1 \right] = \frac{1}{2},$$

and

$$T(R)_C = \text{Tr} \left[\frac{1}{2} \lambda_1 \cdot \frac{1}{2} \lambda_1 \right] = \frac{1}{2},$$

in the normalization of the terms, and where the field strengths are given as:

$$C_A = -\frac{1}{4} \bar{D} \bar{D} e^{-C} D_A e^C, \quad C = \frac{1}{2} g_s \lambda_a C^a, \quad (6.4)$$

$$W_A = -\frac{1}{4} \bar{D} \bar{D} e^{-W} D_A e^W, \quad W = \frac{1}{2} g \sigma_i W^i, \quad (6.5)$$

$$B_A = -\frac{1}{4} \bar{D} \bar{D} D_A B, \quad B = \frac{1}{2} g' B^0. \quad (6.6)$$

6.4 The MSSM superpotential

With the gauge structure of the Standard Model in place we are ready to write down all possible terms in the superpotential. First, we notice that there can be no **tadpole terms** (terms with only one superfield), since there are no superfields that are singlets (zero charge) under all Standard Model gauge groups. The only alternative would be if we introduced right-handed neutrino superfields $\tilde{N}_i$.

For the possible mass terms $m\Phi_i\Phi_j$ we first check the abelian gauge group $U(1)_Y$, where the requirement reduces to the relatively simple sum of hypercharges $Y_i + Y_j = 0$. In Table 6.1 we see that the only possible contributions from the superfields that contain the Standard Model fermions are particle–anti-particle combinations such as $l_{iL}\bar{l}_{iR}$, but these come from superfields with different handedness and cannot be used together.

The exception is a mass term with the two Higgs superfields that have opposite hypercharge $Y = \pm 1$. In order to also be invariant under $SU(2)_L$ we have to write this superpotential term as⁸

$$W_{\text{mass}} = \mu H_u^T i \sigma_2 H_d, \quad (6.7)$$

⁸This is immediately invariant under $SU(3)_C$ since the Higgs fields have no colour charge.

Superfield	L_i	$\bar{E}_i^\dagger$	Q_i	$\bar{U}_i^\dagger$	$\bar{D}_i^\dagger$	$L_i^\dagger$	$\bar{E}_i$	$Q_i^\dagger$	$\bar{U}_i$	$\bar{D}_i$
Fermion	ν_{iL}, l_{iL}	l_{iR}	u_{iL}, d_{iL}	u_{iR}	d_{iR}	$\bar{\nu}_{iR}, \bar{l}_{iR}$	$\bar{l}_{iL}$	$\bar{u}_{iR}, \bar{d}_{iR}$	$\bar{u}_{iL}$	$\bar{d}_{iL}$
Hypercharge	-1	-2	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	1	2	$-\frac{1}{3}$	$-\frac{4}{3}$	$\frac{2}{3}$

Table 6.1: The MSSM superfields with Standard Model fermion content and their hypercharges Y .

where μ is the superpotential mass parameter. This is invariant under $SU(2)_L$ because, with the supergauge transformations $H_d \rightarrow e^{-ig\frac{1}{2}\sigma_k W^k} H_d$ and $H_u \rightarrow H_u e^{-ig\frac{1}{2}\sigma_k^T W^k}$, we get

$$\begin{aligned} H_u^T i\sigma_2 H_d &\rightarrow H_u^T e^{ig\frac{1}{2}\sigma_k^T W^k} i\sigma_2 e^{ig\frac{1}{2}\sigma_k W^k} H_d \\ &= H_u^T i\sigma_2 e^{i\frac{1}{2}g\sigma_k W^k} e^{-i\frac{1}{2}g\sigma_k W^k} H_d = H_u^T i\sigma_2 H_d, \end{aligned}$$

since $\sigma_k^T \sigma_2 = -\sigma_2 \sigma_k$. Usually we ignore the $SU(2)_L$ specific structure and write terms like this as $\mu H_u H_d$, confusing the hell out of anyone that is not used to this convention since we really do mean Eq. (6.7). Notice that if we write (6.7) in terms of component fields we get

$$W_{\text{mass}} = H_u^T i\sigma_2 H_d = H_u^+ H_d^- - H_u^0 H_d^0,$$

which we should have been able to guess because the Lagrangian must also conserve electric charge.

If you have paid very close attention to the argument above you may have noticed that there is one more possibility, namely

$$W_{\text{mass}} = \mu'_i L_i H_u \equiv \mu'_i L_i^T i\sigma_2 H_u = \mu'_i (\nu_i H_u^0 - l_i H_u^+),$$

where μ' is some other mass parameter in the superpotential. This is clearly an allowable term (and we will return to it below), however, it also raises a very interesting question: Could we have $L_i \equiv H_d$? Could the lepton superfields L_i play the rôle of Higgs superfields, thus reducing the field content needed to describe the SM particles in a supersymmetric theory? While not immediately forbidden, this suggestions unfortunately leads to problems with anomaly cancelation, processes with large lepton flavor violation (LFV) and much too massive neutrinos, and has been abandoned.

We have now found all possible mass terms in the superpotential. What about the Yukawa terms? The hypercharge requirement here is $Y_i + Y_j + Y_k = 0$. From our table of hypercharges only the following terms are found to be viable:

$$W_{\text{Yukawa}} = y_{ij}^e L_i H_d E_j + y_{ij}^u Q_i H_u \bar{U}_j + y_{ij}^d Q_i H_d \bar{D}_j + \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k,$$

where we have named and indexed the Yukawa couplings in a natural way.⁹ For all these terms we can simultaneously keep $SU(2)_L$ invariance with the $i\sigma_2$ construction implicitly inserted between any superfield doublets.

For $SU(3)_C$ to be conserved, we need to have colour singlets. Some of these terms are colour singlets by construction since they do not contain any coloured fields. The terms with

⁹For some particular opinion of what is natural.

only two quark superfields contain left-handed Weyl spinors for quarks and anti-quarks, which are $SU(3)_C$ singlets if the superfields come in colour-anti-colour pairs. In representation language they are in the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations of $SU(3)_C$. Written with colour indices explicit we have *e.g.* $L_i Q_j \bar{D}_k = L_i i\sigma_2 Q_j^a \bar{D}_k^a$, where a is the colour index. The final term $\bar{U}_i \bar{D}_j \bar{D}_k$ is a colour singlet once we demand that it is totally anti-symmetric in the colour indices: $\bar{U}_i \bar{D}_j \bar{D}_k \equiv \epsilon_{abc} \bar{U}_i^a \bar{D}_j^b \bar{D}_k^c$.

Our complete superpotential is then:

$$W = \mu H_u H_d + \mu'_i L_i H_u + y_{ij}^e L_i H_d E_j + y_{ij}^u Q_i H_u \bar{U}_j + y_{ij}^d Q_i H_d \bar{D}_j + \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k. \quad (6.8)$$

6.5 R-parity

The superpotential terms LH_u , LLE and $LQ\bar{D}$ that we have written down all violate lepton number conservation, and $\bar{U}\bar{D}\bar{D}$ violates baryon number conservation. Allowing such terms leads to, among other phenomenological problems, processes like proton decay $p \rightarrow e^+ \pi^0$ as shown in Fig. [6.2](#)

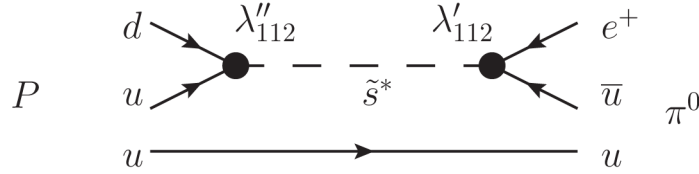


Figure 6.2: Feynman diagram for proton decay with RPV couplings.

We can estimate the resulting proton life-time by noting that the scalar particle (a strange squark $\tilde{s}$) creates an effective Lagrangian term $\lambda \bar{u} \bar{d} e u$ with coupling

$$\lambda = \frac{\lambda'_{112} \lambda''_{112}}{m_{\tilde{s}}^2}, \quad (6.9)$$

where the sparticle mass $m_{\tilde{s}}$ comes from the scalar propagator in the diagram. The resulting matrix element for the process must then be proportional to $|\lambda|^2$. Since the mass scale involved in the problem is the proton mass m_p the phase space integration part of a calculation of the proton decay width must be of the order of m_p^5 . We then have

$$\Gamma_{p \rightarrow e^+ \pi^0} \sim |\lambda|^2 m_p^5 = \frac{|\lambda'_{112} \lambda''_{112}|^2}{m_{\tilde{s}}^4} m_p^5. \quad (6.10)$$

The measured lower limit on the lifetime from watching a lot of protons not decay is $\tau_{p \rightarrow e^+ \pi^0} > 1.6 \cdot 10^{33}$ y or $\tau_{p \rightarrow e^+ \pi^0} > \pi \cdot 10^7$ s/y $\times 1.6 \cdot 10^{33}$ y = $5.0 \cdot 10^{40}$ s, which gives $\Gamma_{p \rightarrow e^+ \pi^0} < 1.3 \cdot 10^{-65}$ GeV, so that with we have the following very strict limit on the combination of two couplings

$$|\lambda'_{112} \lambda''_{112}| < 3.6 \cdot 10^{-26} \left(\frac{m_{\tilde{s}}}{1 \text{ TeV}} \right)^2. \quad (6.11)$$