

For $SU(3)_C$ to be conserved, we need to have colour singlets. Some of these terms are colour singlets by construction since they do not contain any coloured fields. The terms with only two quark superfields contain left-handed Weyl spinors for quarks and anti-quarks, which are $SU(3)_C$ singlets if the superfields come in colour–anti-colour pairs. In representation language they are in the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations of $SU(3)_C$. Written with colour indices explicit we have *e.g.* $L_i Q_j \bar{D}_k = L_i i\sigma_2 Q_j^a \bar{D}_k^a$, where a is the colour index. The final term $\bar{U}_i \bar{D}_j \bar{D}_k$ is a colour singlet once we demand that it is totally anti-symmetric in the colour indices: $\bar{U}_i \bar{D}_j \bar{D}_k \equiv \epsilon_{abc} \bar{U}_i^a \bar{D}_j^b \bar{D}_k^c$. The anti-symmetry property of the Levi-Civita tensor ϵ_{abc} means that we must have $j \neq k$ in λ''_{ijk} .

Our complete superpotential is then:

$$\begin{aligned} W = & \mu H_u H_d + \mu'_i L_i H_u + y_{ij}^e L_i H_d E_j + y_{ij}^u Q_i H_u \bar{U}_j + y_{ij}^d Q_i H_d \bar{D}_j \\ & + \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k. \end{aligned} \quad (6.8)$$

The parameter μ , potentially complex, is a brand new supersymmetric parameter appearing in the superpotential, with no corresponding parameter existing in the Standard Model Lagrangian. However, the Yukawa couplings y_{ij} are identical to the Standard Model Yukawa couplings since they will be required to give mass to the Standard Model fermions after electroweak symmetry breaking when the Higgs fields get a vev, see Section 6.8. The fate of the other parameters will be discussed in the next section.

6.5 R-parity

The superpotential terms LH_u , LLE and $LQ\bar{D}$ that we have written down in (6.8) all violate lepton number conservation, and $\bar{U}\bar{D}\bar{D}$ violates baryon number conservation. Such terms do not exist in the Standard Model, although there is no symmetry forbidding their existence there, instead a combination of what fields exist and the gauge symmetries that limit their interactions means that there are no such tree-level interactions. We call this an **accidental symmetry** of the Standard Model.

Allowing such terms in the MSSM would lead to, among other phenomenological problems, processes like proton decay, for example through $p \rightarrow e^+ \pi^0$ as shown in Fig. 6.2, which breaks both baryon number and lepton number. We can estimate the resulting proton life-time by noting that the exchange of a scalar particle (in this example a strange squark $\tilde{s}$) creates an effective dimension-6 four-fermion interaction Lagrangian term $\lambda \bar{u} \bar{d} e u$ with coupling

$$\lambda = \frac{\lambda'_{112} \lambda''_{112}}{m_{\tilde{s}}^2}, \quad (6.9)$$

where the particle mass $m_{\tilde{s}}$ comes from the scalar propagator in the diagram. The resulting matrix element for the total proton decay process must then be proportional to $|\lambda|^2$, which has mass dimension M^{-4} . Since decay width has mass dimension M , the phase space part of the calculation must provide something of mass dimension M^5 . The only mass scale involved in the problem is the proton mass m_p , thus we approximate the phase space integration part of the proton decay width by m_p^5 . We then have

$$\Gamma_{p \rightarrow e^+ \pi^0} \sim |\lambda|^2 m_p^5 = |\lambda'_{112} \lambda''_{112}|^2 \frac{m_p^5}{m_{\tilde{s}}^4}. \quad (6.10)$$

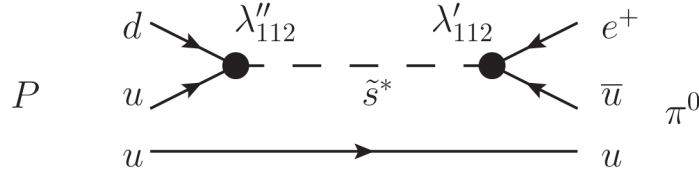


Figure 6.2: Possible Feynman diagram for proton decay with R-parity violating couplings λ''_{112} and λ'_{112} .

The current measured lower limit on the lifetime from watching a lot of protons not decay is $\tau_{p \rightarrow e^+ \pi^0} > 1.6 \cdot 10^{34} \text{ y}$ [11], or, converting to seconds, $\tau_{p \rightarrow e^+ \pi^0} > \pi \cdot 10^7 \text{ s/y} \times 1.6 \cdot 10^{34} \text{ y} = 5.0 \cdot 10^{41} \text{ s}$, which gives a limit on the width

$$\Gamma_{p \rightarrow e^+ \pi^0} = \frac{\hbar}{\tau} < \frac{6.582 \cdot 10^{-25} \text{ GeV s}}{5.0 \cdot 10^{41} \text{ s}} \simeq 1.3 \cdot 10^{-66} \text{ GeV},$$

so that we have the following very strict limit on the combination of the two couplings

$$|\lambda'_{112} \lambda''_{112}| < 1.3 \cdot 10^{-27} \left(\frac{m_{\tilde{s}}}{1 \text{ TeV}} \right)^2. \quad (6.11)$$

Such strict limits can be found on most of the lepton and baryon number couplings using the measured properties of the Standard Model particles, with some exceptions for coupling involving the second and third generation fermions.

To avoid all such lepton and baryon number couplings Fayet (1975) [12] introduced the conservation of R-parity.

Definition: R-parity is a multiplicatively conserved quantum number given by

$$R = (-1)^{2s+3B+L}$$

where s is a particle's spin, B its baryon number and L its lepton number.

For all Standard Model particles, including all the scalar Higgs bosons, this gives $R = 1$, while the superpartners all have $R = -1$. One usually *defines* the MSSM as conserving R-parity. For the MSSM this excludes the terms LH_u , $LL\bar{E}$, $LQ\bar{D}$ and $\bar{U}\bar{D}\bar{D}$ from the superpotential^[10] leaving us with the R-parity conserving superpotential

$$W = \mu H_u H_d + y_{ij}^e L_i H_d E_j + y_{ij}^u Q_i H_u \bar{U}_j + y_{ij}^d Q_i H_d \bar{D}_j. \quad (6.12)$$

The consequence of this somewhat *ad hoc* definition is that in all interactions the total number of incoming and outgoing supersymmetric particles must be an even number $2n$, so that the total R-number is $(-1)^{2n} = 1$. This leads to the following very important phenomenological consequences:

¹⁰All the superpotential Yukawa terms lead to component field terms of the form $A_i \psi_j \psi_k$. If the scalar A_i here is *not* a Higgs boson, then it is a superpartner and if none of the fermions come from a Higgs superfield so that they are also a superpartner the term breaks R-parity conservation. The superpotential mass terms have component field terms of the form $\psi_i \psi_j$. If one of the fermions here comes from a Higgs superfield, then it is a superpartner, and if the other does not, the term breaks R-parity.

1. All sparticles must be produced in pairs (ignoring the very low probability of producing four or more).
2. Sparticles must annihilate in pairs.
3. The **lightest supersymmetric particle** (LSP) is absolutely stable, and every other sparticle must decay down to the LSP (possibly in multiple steps).

6.6 Supersymmetry breaking terms in the MSSM

We can directly apply our previous arguments on gauge invariance, that we used when discussing the superpotential, on the general soft-breaking terms in Eq. (5.7) in order to determine which supersymmetry breaking terms are allowed in the MSSM, keeping also in mind the requirement of R-parity conservation.

Mass terms of the form

$$-\frac{1}{4T(R)}M\theta\theta\bar{\theta}\bar{\theta}\text{Tr}[W^AW_A] + \text{h.c.},$$

are allowed because they have the same gauge structure as the supersymmetric field strength terms. In component fields only the fermions in the vector superfields survive, and are for the MSSM:

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2}M_1\tilde{B}\tilde{B} - \frac{1}{2}M_2\tilde{W}^i\tilde{W}^i - \frac{1}{2}M_3\tilde{g}^a\tilde{g}^a + \text{c.c.},$$

where the M_i are potentially complex-valued. This gives six new parameters.

Yukawa terms

$$-\frac{1}{6}a_{ijk}\theta\theta\bar{\theta}\bar{\theta}\Phi_i\Phi_j\Phi_k + \text{h.c.},$$

are allowed when a corresponding term exist in the superpotential – meaning they are gauge invariant. In component fields only the scalar parts of the superfields survive, and the allowed terms are

$$\mathcal{L}_{\text{soft}} = -a_{ij}^e\tilde{L}_iH_d\tilde{l}_{jR}^* - a_{ij}^u\tilde{Q}_iH_u\tilde{u}_{jR}^* - a_{ij}^d\tilde{Q}_iH_d\tilde{d}_{jR}^* + \text{c.c.},$$

where we remind you that the H here refers to scalar parts of the Higgs superfields. We can see that all of these terms are R-parity conserving, since they consist of two sparticles and one (Higgs) particle. The couplings a_{ij} are all potentially complex valued, so this gives us 54 new parameters.

The mass terms

$$-\frac{1}{2}b_{ij}\theta\theta\bar{\theta}\bar{\theta}\Phi_i\Phi_j + \text{h.c.},$$

are again only allowed for corresponding terms in the superpotential, *i.e.*

$$\mathcal{L}_{\text{soft}} = -bH_uH_d + \text{c.c.},$$

where b is potentially complex valued, which gives us 2 new parameters.^[11] Tadpole terms are not allowed, as there are no tadpoles in the superpotential.

Mass terms

$$-m_{ij}^2\theta\theta\bar{\theta}\bar{\theta}\Phi_i^\dagger\Phi_j,$$

¹¹The coupling b is sometimes written $B\mu$ where B is a unitless constant that indicates how different the coupling is from the corresponding coupling in the superpotential.

are allowed because they have the same gauge structure as the supersymmetric kinetic terms. In component fields again only the scalar fields survive, and in the MSSM they are:

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -(m_{ij}^L)^2 \tilde{L}_i^\dagger \tilde{L}_j - (m_{ij}^e)^2 \tilde{l}_{iR}^* \tilde{l}_{jR} - (m_{ij}^Q)^2 \tilde{Q}_i^\dagger \tilde{Q}_j - (m_{ij}^u)^2 \tilde{u}_{iR}^* \tilde{u}_{jR} - (m_{ij}^d)^2 \tilde{d}_{iR}^* \tilde{d}_{jR} \\ & - m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d,\end{aligned}\quad (6.13)$$

where the m_{ij}^2 are complex valued, however, also hermetic. This gives rise to 47 new parameters. Despite technically being allowed the MSSM ignores the “maybe-soft” terms in Eq. (5.8).

In total, after using our freedom to choose our basis for the fields wisely in order to remove what freedom we can, the MSSM has 105 new parameters compared to the Standard Model, 104 of these are soft-breaking terms and μ is the only new parameter in the superpotential.

6.7 Renormalisation group equations

Renormalisation, the removal of infinities from field theory predictions, introduces a fixed scale μ at which the parameters of the Lagrangian, the couplings, are defined. For example, the charge of the electron is not simply the bare charge e , but a charge at a given energy scale μ , $e(\mu)$, which is the scale at which the theory describes the electron, and which we can measure in an experiment at that scale. Scattering an electron at very high energy will require a different value of $e(\mu)$ than at a low energy. This is an experimentally well verified fact.¹²

However, since μ is not an observable *per se* but in principle a choice of how to write down the theory (at which energy to write down the Lagrangian), the action S should be invariant under a change of μ , which is expressed as:

$$\mu \frac{d}{d\mu} S(\Phi, \lambda, \mu) = 0, \quad (6.14)$$

where λ is a generic name for the couplings of the theory and Φ represents the (super)fields that have been renormalised.¹³ This equation can be re-written in terms of partial derivatives

$$\left(\mu \frac{\partial}{\partial \mu} + \mu \frac{\partial \lambda}{\partial \mu} \frac{\partial}{\partial \lambda} \right) S(\Phi, \lambda, \mu) = 0, \quad (6.15)$$

which is the **renormalisation group equation** (RGE).

We can look at the behaviour of a Lagrangian parameter λ as a function of the energy scale μ away from the value where it was defined, often denoted μ_0 . This is controlled by the **β -function**:

$$\beta_\lambda \equiv \mu \frac{\partial \lambda}{\partial \mu}. \quad (6.16)$$

These β -functions can be found from the so-called counterterm Z that renormalises the parameter. As an example, take a **gauge coupling constant** g_0 defined (taken from measurement)

¹²It is also impossible to avoid if we accept that the electron is a point particle. Since the potential has the form $V(r) \propto e/r$ an infinite energy would appear unless we were somehow to modify the charge at high energies, or equivalently, short distances.

¹³In Sec. 4.8 we mentioned how the non-renormalisation theorem implies that we do not need to renormalise the coupling constants of the superpotential separately.

at some scale μ_0 . This is given by (in $d = 4 - \epsilon$ dimensions):¹⁴

$$g_0 = Zg\mu^{-\epsilon/2}.$$

Then, differentiating both sides with respect to μ ,

$$\begin{aligned} 0 &= \frac{\partial Z}{\partial \mu} g\mu^{-\epsilon/2} + Z \frac{\partial g}{\partial \mu} \mu^{-\epsilon/2} - \frac{\epsilon}{2} Zg\mu^{-\epsilon/2-1} \\ \mu \frac{\partial g}{\partial \mu} &= \frac{\epsilon}{2} g - \frac{g\mu}{Z} \frac{\partial Z}{\partial \mu} \\ \mu \frac{\partial g}{\partial \mu} &= \frac{\epsilon}{2} g - g\mu \frac{\partial}{\partial \mu} \ln Z, \end{aligned}$$

and taking the limit $\epsilon \rightarrow 0$:

$$\beta_g = \mu \frac{\partial g}{\partial \mu} = -g\gamma_g,$$

where we have defined the **anomalous dimension** of g

$$\gamma_g = \mu \frac{\partial}{\partial \mu} \ln Z. \quad (6.17)$$

The reason for keeping around the factor of μ in the definition of the β -function is that it typically changes very slowly over large differences in energy scale, so it is practical to change variable to $t = \ln \mu$, so that $\mu \frac{\partial}{\partial \mu} = \frac{\partial}{\partial t}$ and $\beta_g = \frac{\partial g}{\partial t}$.

Z can now be calculated to the required loop-order to find the β -function to that order, and in turn the running of the coupling constant with μ . By evaluating to one-loop order we can find that for our particular example of a gauge coupling constant

$$\gamma_g|_{1\text{-loop}} = -\frac{1}{16\pi^2} g^2 \left(\sum_R T(R) - 3C(A) \right), \quad (6.18)$$

where the sum of Dynkin indices is over all superfields that transform under a representation R of the gauge group in question, and $C(A)$ is the **quadratic Casimir invariant** of the representation A of the vector field under the gauge group

$$C(A)\delta_{ij} = (T^a T^a)_{ij}.$$

For the adjoint representation of $U(1)$ this is 0, and for $SU(N)$ this is N . The running of the coupling constants is particularly important since it will later lead us to the concept of gauge coupling unification.

6.8 Radiative EWSB

In the SM the vector bosons are given mass spontaneous by electroweak symmetry breaking (EWSB), which is induced by the shape of the scalar potential for a scalar field Φ :

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad (6.19)$$

¹⁴The factor $\mu^{-\epsilon/2}$ is there to ensure that the scale of g_0 is correct.